ETH
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Distributed Computing

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Distributed Systems Part II

Solution to Exercise Sheet 5

1 The Resilience of a Quorum System

a) No such quorum system exists. According to the definition of a quorum system, every two quorum of a quorum system intersect. So at least one server is part of both quorums. The fact that all servers of a particular quorum fail, implies that in each other quorum at least one server fails, namely the one which lies in the intersection. Therefore it is not possible to achieve a quorum anymore and the quorum system does not work anymore.

b) Just 1 - as soon as 2 servers fail, no quorum survives.

c) Imagine a quorum system where you have one quorum of size 1, and all remaining quorums are the elements of the powerset of the remaining $n-1$ nodes, each joined with the first mentioned server. This gives 2^{n-1} quorums. Can there be more? No! Consider a set from the powerset of n servers. Its complement cannot be a quorum as well, as they don't overlap. This gives an upper bound of $2^n/2 = 2^{n-1}$.

2 A Quorum System

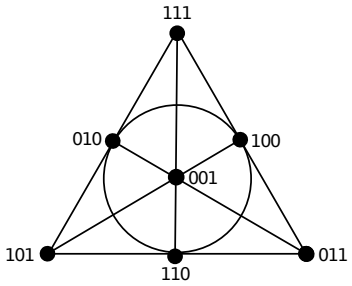

Figure 1: Quorum System

a) This quorum system consists of 7 quorums. As work is defined as the maximum number of servers in a quorum, its work is 3. The best access strategy consists of uniform accessing each quorum. So its load is 3/7.

b) Its resilience $R(Q) = 2$. If for example the nodes 101, 010 and 111 fail, no other quorum can be achieved.

3 S-Uniform Quorum Systems

Definitions:

S-uniform: A quorum system Q is *s-uniform* if every quorum in Q has exactly s elements.
Balanced access strategy: An access strategy W for a quorum system Q is *balanced* if it satisfies $l_W(i) = L$ for all $P_i \in P$.

Claim: An s-uniform quorum system Q reaches an optimal load with a balanced access strategy.

a) In an s-uniform quorum system each quorum has exactly s elements. So independently which quorum is accessed, s servers have to work. Summed up over all servers we reach a load of s. As the load of a quorum system is defined as the maximal load of any server, the best strategy is to evenly distribute this load on the servers.

b) Let $P = \{p_1, p_2, ..., p_n\}$ be the set of servers and $Q = \{q_1, q_2, ..., q_m\}$ an s-uniform quorum system on P. W is a strategy, therefore it holds that: $\sum_{i=1}^{m} w_i = 1$ Furthermore let $l_w(p_i) = \sum_{q \in Q; p_i \in q} P_w(q)$ be the load of server p_i with strategy w.
Then it holds that:

$$\sum_{p_i \in P} \sum_{p_i \in q_j} w_j = \sum_{j=1}^{m} w_j \sum_{p_i \in q_j} 1 = s \sum_{j=1}^{m} w_j = s$$

To minimize the maximal load on any server the optimal strategy is to evenly distribute this load on all servers. This leads to a balanced system load of s/n.