Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Networked Systems Group (NSG)

HS 2023

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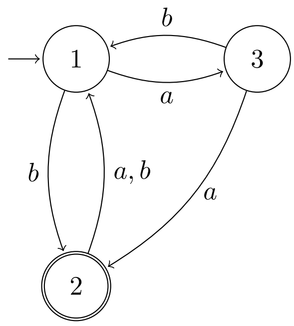
based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 3

1 From DFA to Regular Expression

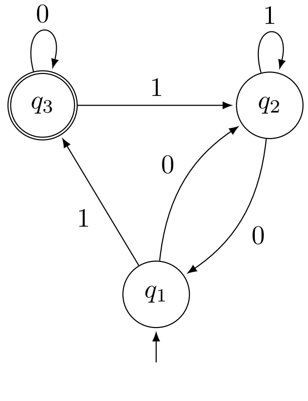
Build a language-equivalent GNFA for the DFA below. Then, as seen during the lecture, eliminate the intermediate states of the GNFA and derive the regular expression that represents the language accepted by the DFA.



Remark: The sample solution rips out the states in the order 2 – 3 – 1. For further training, you can consider other orderings and see how the resulting REX looks slightly different but captures the same language!

2 Transforming Automata [Exam HS14]

Consider the following DFA over the alphabet $\Sigma = \{0, 1\}$. Give a regular expression for the language L accepted by the following automaton. If you like, you can do this by ripping out states as presented in the lecture.



3 Pumping Lemma

Which of the following languages is regular? Prove your claims!

a) $L_1 = \{1^n 0 2^n \mid n \geq 0\}$

b) $L_2 = \{0^a 1^b 0^c 1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\}$

4 Pumping Lemma Revisited

a) Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

b) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number p to determine the number of states of a minimal DFA accepting L? What about the number of states of the corresponding NFA?

5 Minimum Pumping Length

Consider the regular language $L = 1^*0^+1^+0^* \cup 111^+0^+$. Give the minimum pumping length and briefly explain the intuition behind your answer.

6 The art of being regular

Assume that the alphabet Σ is $\{0, 1, \#\}$ and consider the language $L = \{x\#y \mid x + y = 3y\}$ in which x and y are unsigned binary numbers. For instance, the string 1000#100 belongs to L. Is L regular?
If so, exhibit a finite automaton (deterministic or not) or a regular expression recognizing it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

Bonus tasks:

- Determine whether $L = \{x\#y \mid x + y = 3y\}$ is context-free.
- Show whether $L' = \{x\#y \mid x + \text{reverse}(y) = 3 \cdot \text{reverse}(y)\}$ is context-free.
  The reverse() *-function takes an integer as a bitstring and reverses the order of its bits.*