1 Comparison of Finite Automata

Here are two simple finite automata:

For each, we have a one bit encoding for the states \((x_A, x_B)\), one binary output \((y_A, y_B)\), and one common binary input \((u)\). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

\(\text{a) Express the characteristic function of the transition relation for both automaton, } \psi_T(x, x', u).\)

\(\text{b) Express the joint transition function, } \psi_f.\)

\(\text{Reminder: } \psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u)).\)

\(\text{c) Express the characteristic function of the reachable states, } \psi_X(x_A, x_B).\)

\(\text{d) Express the characteristic function of the reachable output, } \psi_Y(y_A, y_B).\)

\(\text{e) Are the two automata equivalent? } \text{Hint: } \text{Evaluate, for example, } \psi_Y(0, 1).\)
2 Temporal Logic

a) We consider the following automaton. The property $a$ is true on the colored states (0 and 3).

For each of the following CTL formula, list all the states for which it holds true.

(i) EF $a$
(ii) EG $a$
(iii) EX AX $a$
(iv) EF ( $a$ AND EX NOT($a$) )

b) Given the transition function $\psi_f(x, x')$ and the characteristic function $\psi_Z(x)$ for a set $Z$, write a small pseudo-code which returns the characteristic function of $\psi_{AFZ}(x)$. It can be expressed as symbolic boolean functions, like $x_A x_B x'_B + x_A x'_B x'_B$.

Hint: To do this, simply use the classic boolean operators AND, OR, NOT and ! =. You can also use the operator $\text{PRE}(Q, f)$, which returns the predecessor of the set $Q$ by the transition function $f$. That is,

$$\text{PRE}(Q, f) = \{ q' : \exists x, \psi_f(q', q) \cdot \psi_Q(q) = 1 \}$$

Hint: It can be useful to reformulate AFZ as another CTL formula.