# Brief Announcement: Distributed Contention Resolution in Wireless Networks[*]


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## ABSTRACT

We present and analyze simple distributed contention resolution protocols for wireless networks. In our setting, one is given $n$ pairs of senders and receivers located in a metric space. Each sender wants to transmit a signal to its receiver at a prespecified power level, e. g., all senders use the same, uniform power level as it is typically implemented in practice. Our analysis is based on the physical model in which the success of a transmission depends on the Signal-to-Interference-plus-Noise-Ratio (SINR). The objective is to minimize the number of time slots until all signals are successfully transmitted.

Our main technical contribution is the introduction of a measure called maximum average affectance enabling us to analyze random contention-resolution algorithms in which each packet is transmitted in each step with a fixed probability depending on the maximum average affectance. We prove that the schedule generated this way is only an $\mathcal{O}(\log^2 n)$ factor longer than the optimal one, provided that the prespecified power levels satisfy natural monontonicity properties. By modifying the algorithm, senders need not to know the maximum average affectance in advance but only static information about the network. In addition, we extend our approach to multi-hop communication achieving the same appoximation factor.



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**General Terms:** Algorithms, Theory

**Keywords:** Wireless Network, Interference, Physical Model, SINR, Distributed Scheduling


## 1. INTRODUCTION

We analyze distributed contention-resolutions protocols for packet scheduling in wireless networks giving worst-case guarantees. The interference constraints are modelled by the *physical interference model* [3]. Between any two nodes of the network $u$ and $v$ a distance $d(u, v)$ is defined. If node $u$ transmits a signal at power level $p$ then it is received by $v$ with strength $p/d(u,v)^\alpha$, where the constant $\alpha > 0$ is the so-called *path-loss exponent*[1]. The node $v$ can successfully decode this signal if the signal strength received from the intended sender is at least $\beta$ times as large as the signals strengths by interfering transmissions made at the same time plus ambient noise. This is, the *Signal-to-Interference-plus-Noise Ratio (SINR)* is above some threshold $\beta \geq 0$, the so-called *gain*.

In our setting, we are given a set of $n$ requests $\mathcal{R} \subseteq V \times V$, corresponding to pairs of nodes from a metric space and a power level $p(\ell) > 0$ for each of them. We have to select a time slot $c(\ell) \in \{1, \ldots, k\}$ for each request $\ell \in \mathcal{R}$ such that for each $\ell = (u, v) \in \mathcal{R}$ the SINR constraint

$$\frac{p(\ell)}{d(u,v)^\alpha} \geq \beta \left( \sum_{\substack{\ell'=(u',v') \in \mathcal{R} \\ c(\ell)=c(\ell')}} \frac{p(\ell')}{d(u',v)^\alpha} + N \right)$$

is fulfilled. The constant $N \geq 0$ expresses ambient noise that all transmissions have to cope with. The objective is to minimize the number of time slots $k$.

Our objective is to calculate a schedule whose length is close to the optimal schedule length that could possibly be achieved by an optimal schedule in the same instance. We denote the optimal schedule length for $\mathcal{R}$ that uses some fixed power assignment $p$ by $T(\mathcal{R}, p)$. For the problem variant in which powers are subject to optimization, a similar measure has been introduced by Moscibroda et al. [6] as *scheduling complexity* $T(\mathcal{R})$.

Powers might be given by hardware or by a scheme. Such schemes for assigning the powers that have been used in related work include *uniform* [5], *linear* [2] and *square-root* (or mean) power assignments [1, 4]. For each of them, there are specialized algorithms, which are mostly centralized. So far, de-centralized algorithms with a provable performance guarantee are only known for linear power assignments [2]. Furthermore, most existing transceivers support only a relatively small, fixed number of possible power levels so that a practical implementation of both linear and square-root power assignments remains a challenge. As a consequence it is necessary to have more general algorithms which not only work for a certain power scheme.

Our algorithms do not require a certain power scheme but work for every power assignment satisfying the following natural conditions. First, it has to be *non-decreasing* and *sublinear*. That means if $d(\ell) \leq d(\ell')$ for two requests $\ell, \ell' \in \mathcal{R}$ then $p(\ell) \leq p(\ell')$ and $p(\ell)/d(\ell)^\alpha \geq p(\ell')/d(\ell')^\alpha$. So the


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[1]Typically it is assumed that $2 < \alpha < 5$. However, our analysis works for any $\alpha > 0$.

transmission power of $\ell'$ has to be as least as large as the one for $\ell$. At the same time, the received power at the receiver of $\ell'$ must not be larger than the one at the receiver of $\ell$. This monotonicity condition is very natural and is fulfilled by all previously studied power assignments, particularly the ones mentioned above. The second condition is that powers are chosen sufficiently large so that ambient noise plays a minor part compared to interference.

## 2. MAXIMUM AVERAGE AFFECTANCE

We introduce a new measure called *maximum average affectance* $\bar{A}$ that depends on the request set $\mathcal{R}$ and the power assignment $p$. This measure extends a so-called *measure of interference* for linear power assignments [2] in a non-trivial way towards general power assignments satisfying the above conditions.

For two requests $\ell = (u, v)$ and $\ell' = (u', v')$, and a power assignment $p$, we define the *affectance* of $\ell$ on $\ell'$ by

$$a_p(\ell, \ell') = \min\left\{1, \beta \frac{p(\ell)}{d(u, v')^\alpha} \bigg/ \left(\frac{p(\ell')}{d(u', v')^\alpha} - \beta N\right)\right\} .$$

The notion of affectance was introduced by Halldórsson and Wattenhofer [5], which we extended to arbitrary power assignments and bounded by 1. When taking the noise out of consideration, it indicates which amount of interference $\ell$ induces at $\ell'$, normalized by the signal strength from the intended sender of $\ell$. As a consequence the sum of affectance is at most 1 for a request set that may be assigned to same time slot.

To get the *maximum average affectance* $\bar{A}(\mathcal{R}, p)$, we take the maximum over all subsets of requests and consider the average affectance a link is exposed to from all other requests in this subset:

$$\bar{A} = \max_{M \subseteq \mathcal{R}} \operatorname*{avg}_{\ell' \in M} \sum_{\ell \in M} a_p(\ell, \ell') = \max_{M \subseteq \mathcal{R}} \frac{1}{|M|} \sum_{\ell' \in M} \sum_{\ell \in M} a_p(\ell, \ell') .$$

The *maximum average affectance* is the key to analyze random contention-resolution based algorithms and comparing the perfomance to the optimum. In our basic algorithm each sender transmits with a certain probability $q$ in each step until one of the transmissions has successfully been received. We first prove that if $q \leq 1/4\bar{A}$ all transmissions are successful within $\mathcal{O}(\log n/q)$ time slots whp[2]. Thus choosing $q = 1/4\bar{A}$, we generate a schedule of length $\mathcal{O}(\bar{A} \cdot \log n)$ whp. We complement this result by proving $\bar{A}$ is at most a factor $\mathcal{O}(\log n)$ larger than the optimal schedule length $T(\mathcal{R}, p)$. In combination, this yields the schedule generated by the algorithm has length $\mathcal{O}(T(\mathcal{R}, p) \cdot \log^2 n)$.

## 3. TOWARDS DISTRIBUTED ALGORITHMS

An algorithm that is applicable in a realistic environment has to work in a distributed fashion with as few information as possible. In order to achieve this goal, we present two modifications. These do not affect the schedule length vitally and we still get schedules of length $\mathcal{O}(\bar{A} \cdot \log n)$ whp. On the one hand, we extend it such that the network nodes do not have to know $\bar{A}$ anymore but adapt the transmission probability $q$ on their own. On the other hand, we present a way to inform each sender if a transmission has successfully been received by transmitting acknowledgement packets. This is not a trivial task because these acknowledgement packets may also interfere.

Altogether, this is the first distributed algorithm to the interference scheduling problem with a guaranteed approximation ratio. It requires only static information on the network that can be spread at the time of deployment. Particularly, the number of network nodes, the clock synchronization and the power assignment can be seen as such static information. In contrast, no information about the current state of the network will be necessary. For example, communication requests arise after the deployment and an algorithm has to work without knowledge on which requests have to be served by the network and which of them were already successfully served. Our algorithm can be run on all senders and receivers of a network such that during the execution no central entity is needed.

As a further result, we adapt the ideas to a distributed multi-hop algorithm that allows packets to use intermediate relay nodes. For a fixed choice of paths and powers we get an $\mathcal{O}(\log^2 n)$ whp approximation for this problem as well.

[2]with high probability: with probability $1 - n^c$ for each constant $c$

## 4. DISCUSSION AND OPEN PROBLEMS

While previous algorithms are mostly centralized, the algorithms and analyses we present seem to be much closer to realistic scenarios as the scheduling protocol only needs static information. Nevertheless, it is an interesting question which performance can still be achieved without this knowledge. Unfortunately, we cannot get rid of any of these assumptions in a non-trivial way. However, concerning the number of nodes and the clock synchronization there are various results in other scenarios that could possibly transferred.

For the power assignment problem the best solution up to know is to take distance-based power schemes such as the square-root power assignment. Up to now there is no known way to calculate a power assignment achieving an approximation ratio that is close to optimal in all instances, even not in a centralized way. This leaves much space for future research.

## 5. ACKNOWLEDGEMENTS

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