ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing With Imperfect Randomness

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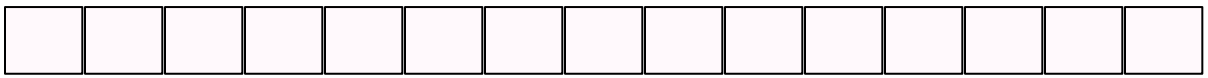
Presentation by: Daniel Thomas

1. Randomness

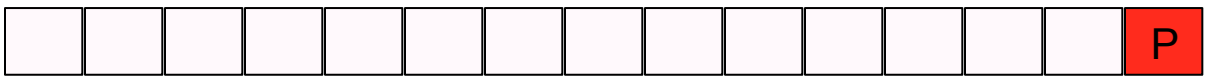
1. Randomness

QuickSort



1. Randomness

QuickSort



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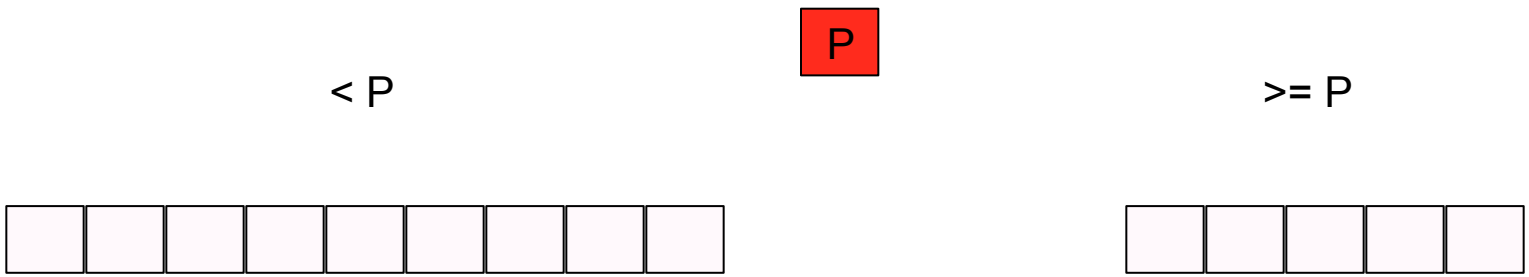
QuickSort



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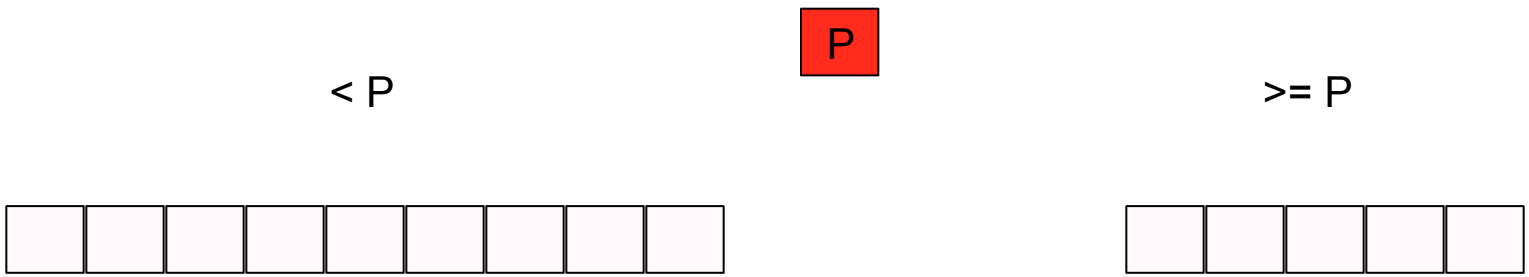
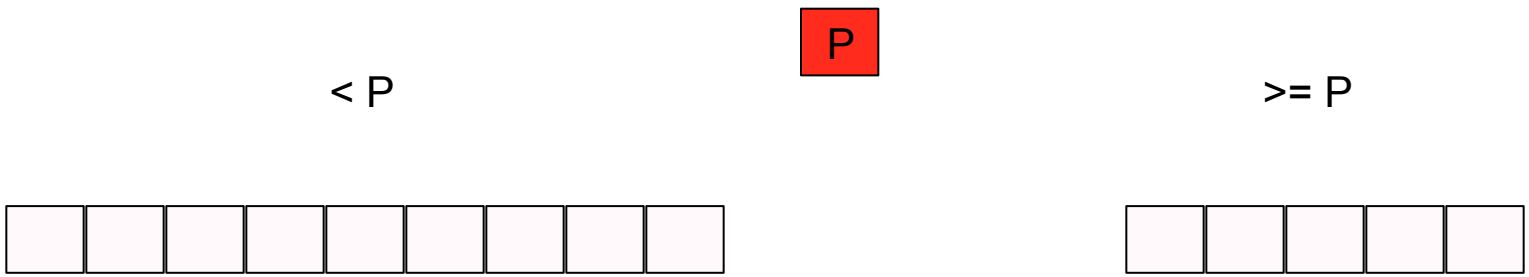
QuickSort



What If Already Sorted?

1. Randomness

QuickSort



What If Already Sorted?

→ Randomized QuickSort

1. Randomness

Real Randomness?
(Strong Randomness)

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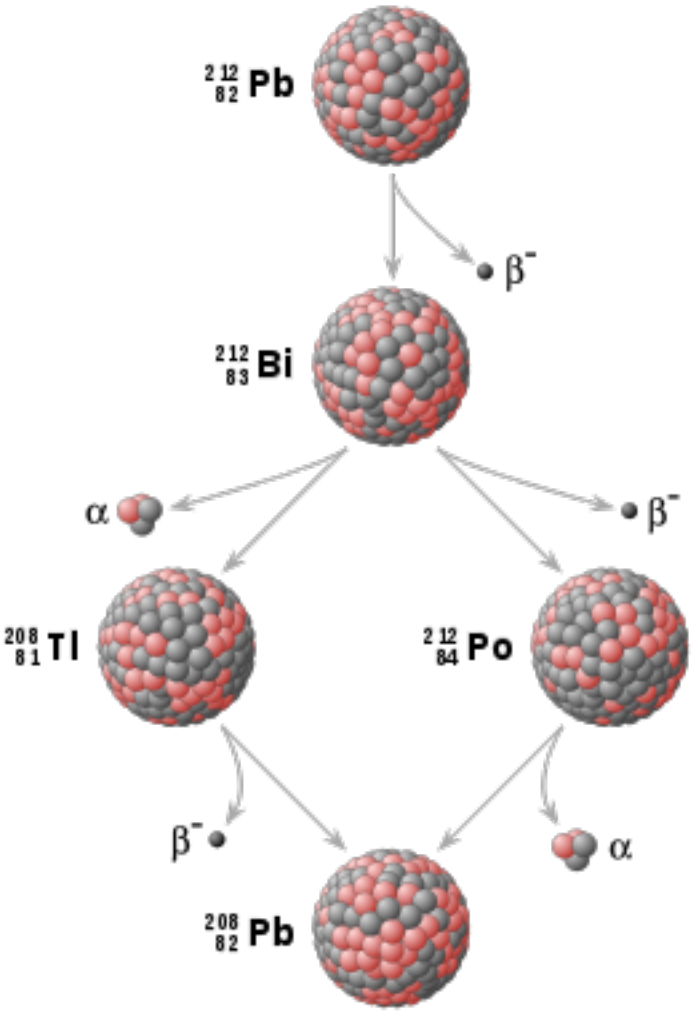
(Strong Randomness)

Atomic Decay?



1. Random

Real Ran
(Strong R

Atomic D

1. Randomness

Pseudorandomness?
(Weak Randmoness?)

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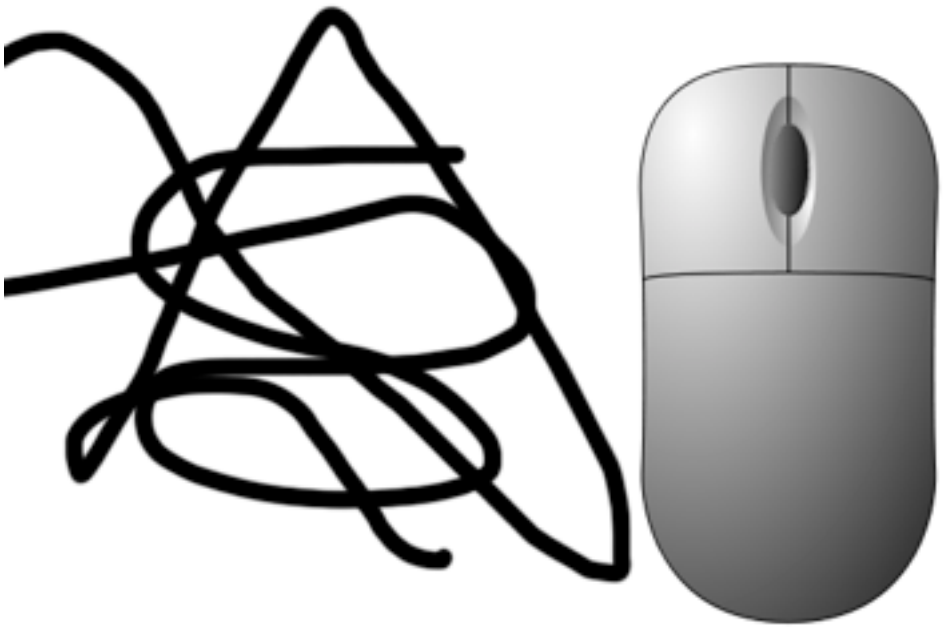
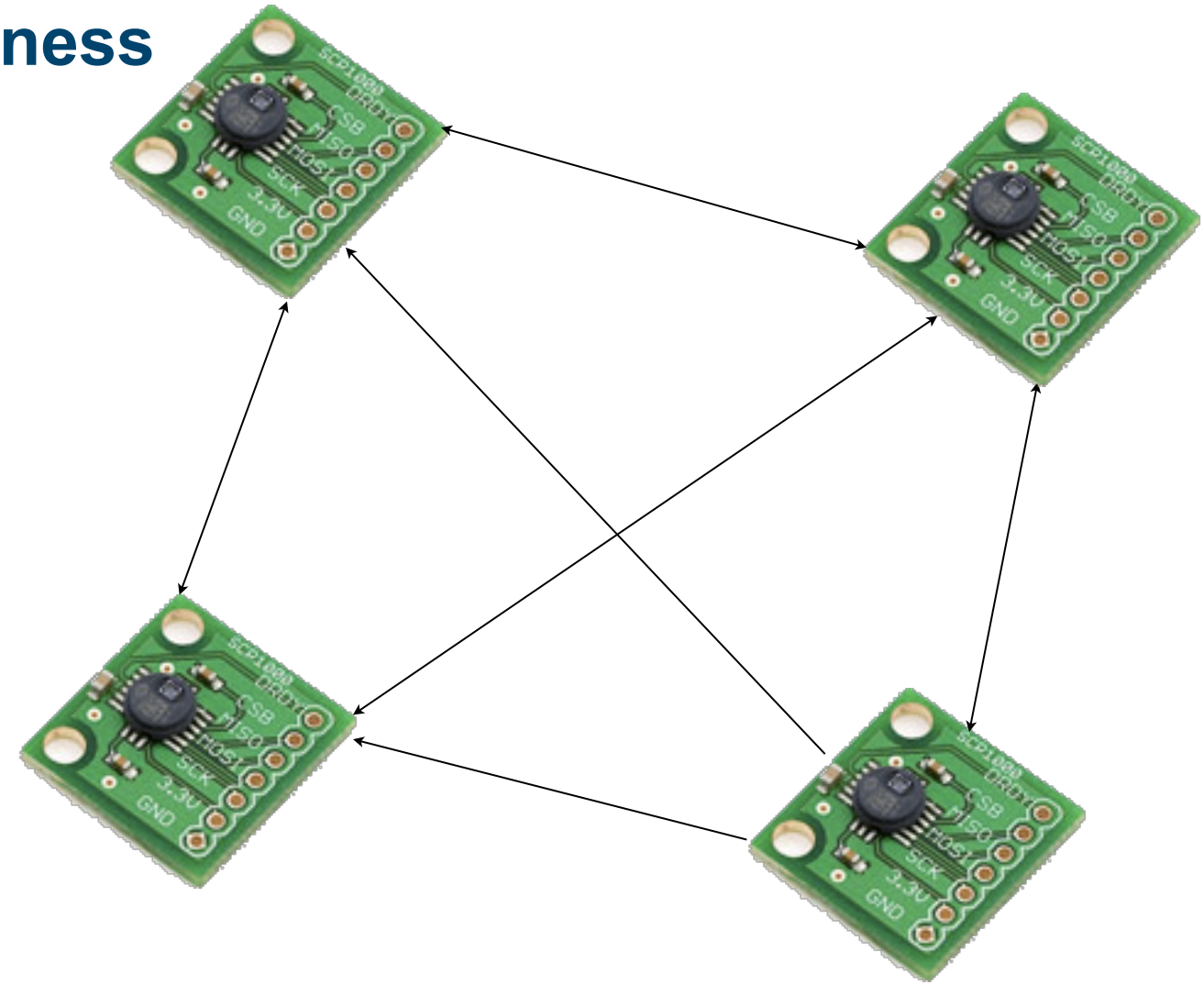
Pseudorandomness?
(Weak Randmoness?)

Get Randomness From System?



1. Randomness

Setting



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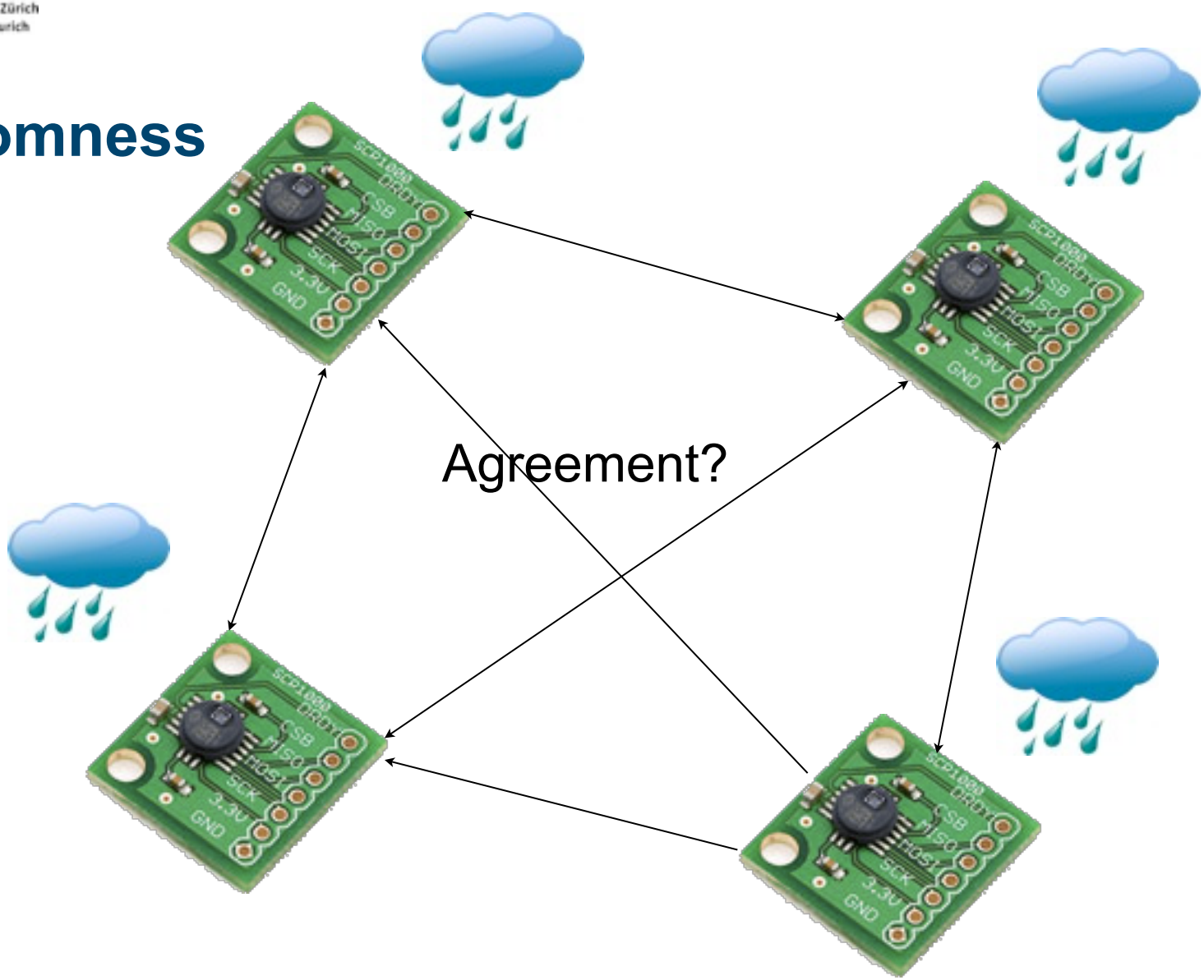
Setting



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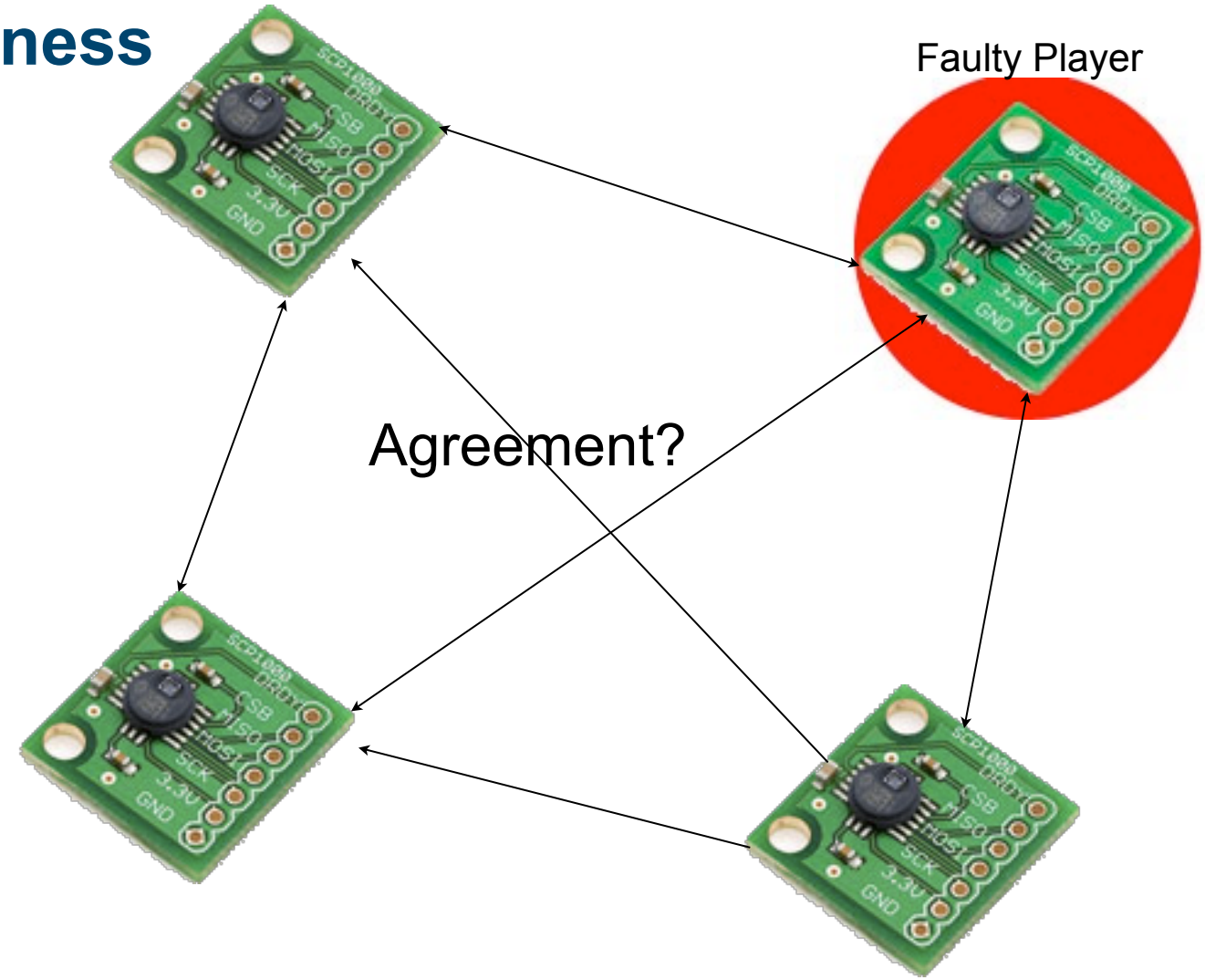
Setting



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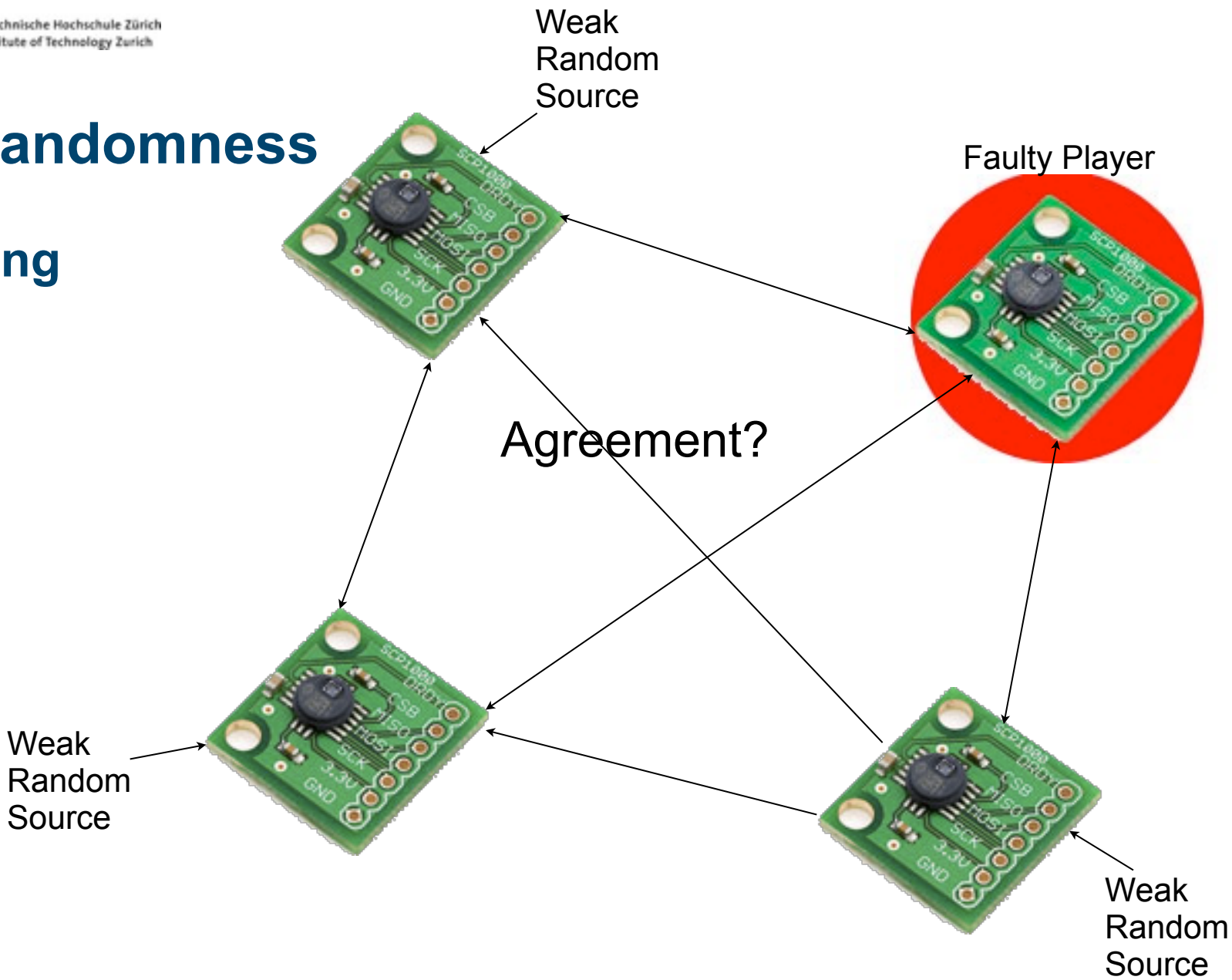
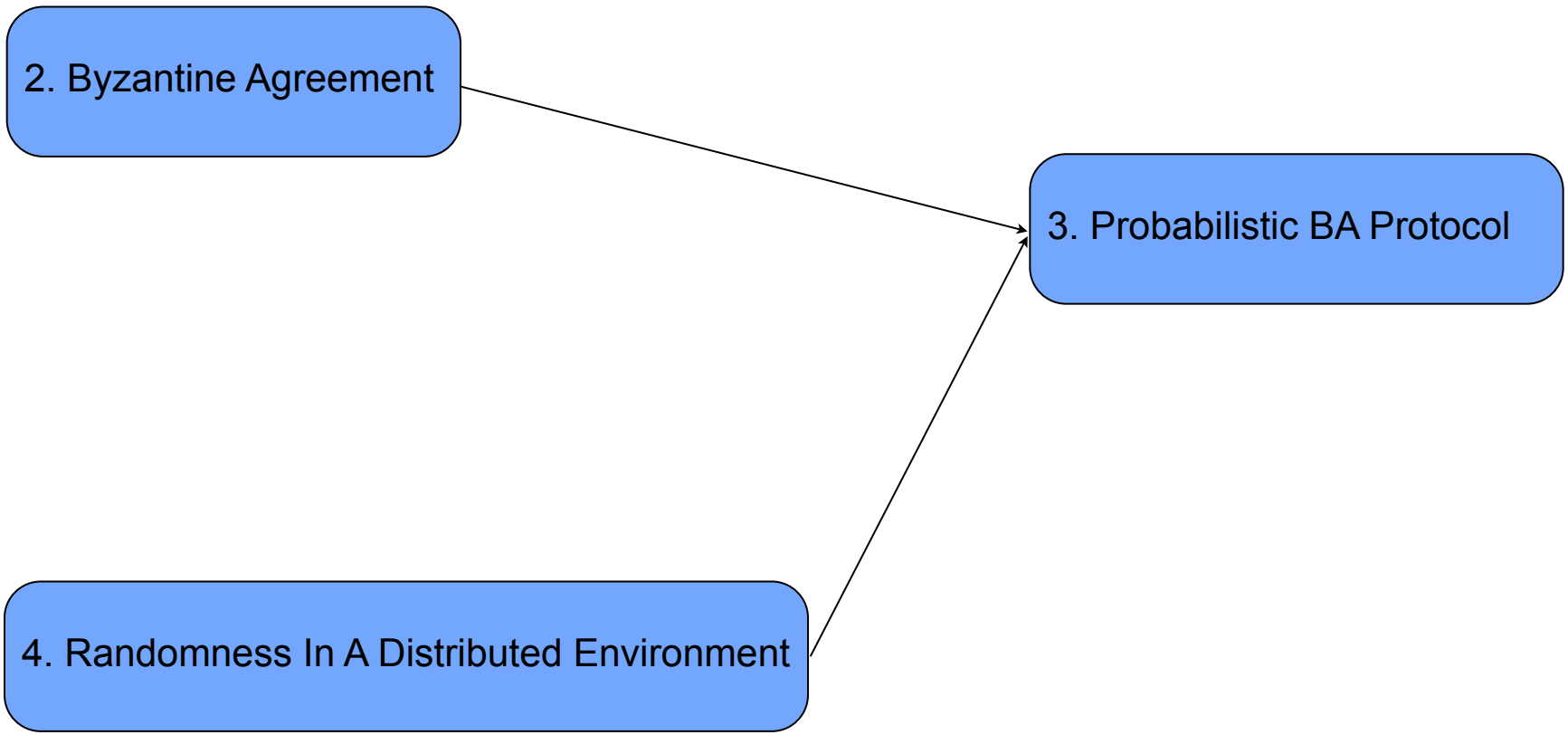
Setting



1. Randomness, Overview



2. Byzantine Agreement (single fault case)

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A. The non-faulty generals compute exactly the same vector

B. The element of this vector corresponding to a given non-faulty general is the private value/ opinion of this general

2. Byzantine Agreement (single fault case)

Works if n >= 3m + 1
And for m+1 rounds are required.

3. Probabilistic Byzantine Agreement Protocols

Deterministic Algorithm	Randomized Algorithm A Simple and Efficient Randomized Byzantine Agreement
m+1 rounds	O(m/log n) rounds

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Randomized Algorithm Expects Players To Have Real Random Source!

4. Randomness In A Distributed Environment

Formalizing Our Randomness Model

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Formalizing Our Randomness Model

- Strong Randomness: U_m
- Weak Randomness: (k, δ)-weak source
- No Randomness

4. Randomness In A Distributed Environment

Randomness Model

- Each player (sensor) has access to its own weak source that is independent of the sources of all other players

- Is there a way to get real randomness (a string of unbiased and independent random bits) by combining weak random sources?

4. Randomness In A Distributed Environment

Randomness Model

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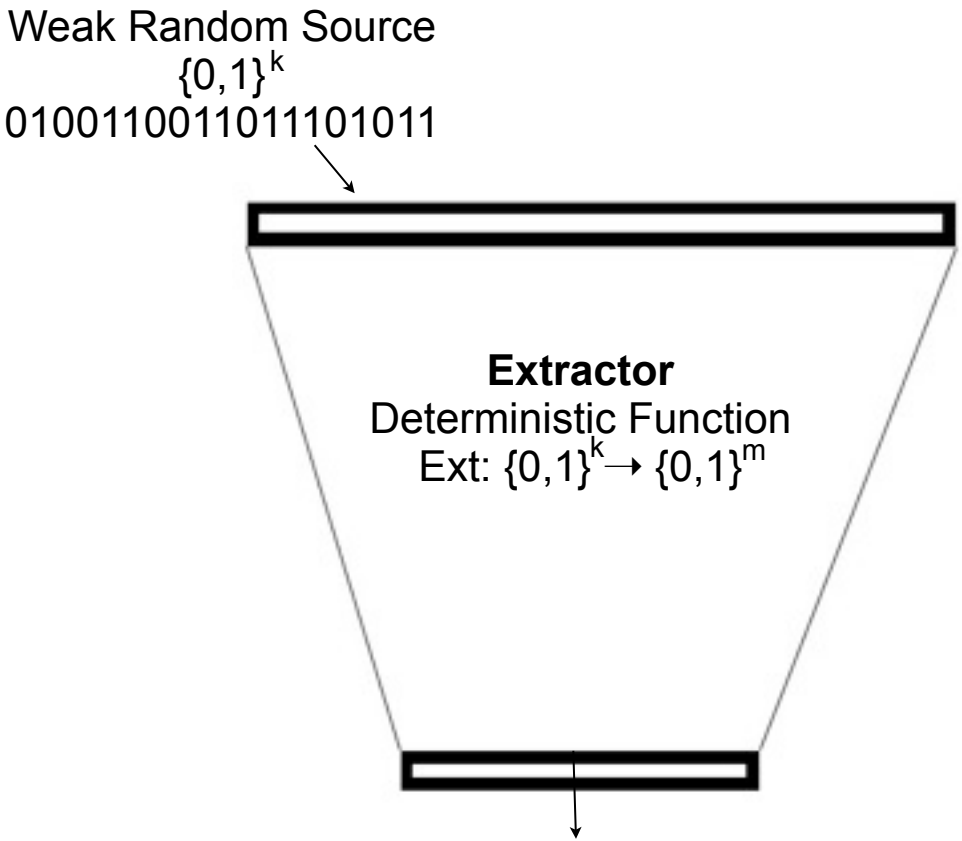
- Is there a way to get real randomness (a string of unbiased and independent random bits) by combining weak random sources?

Extractors!

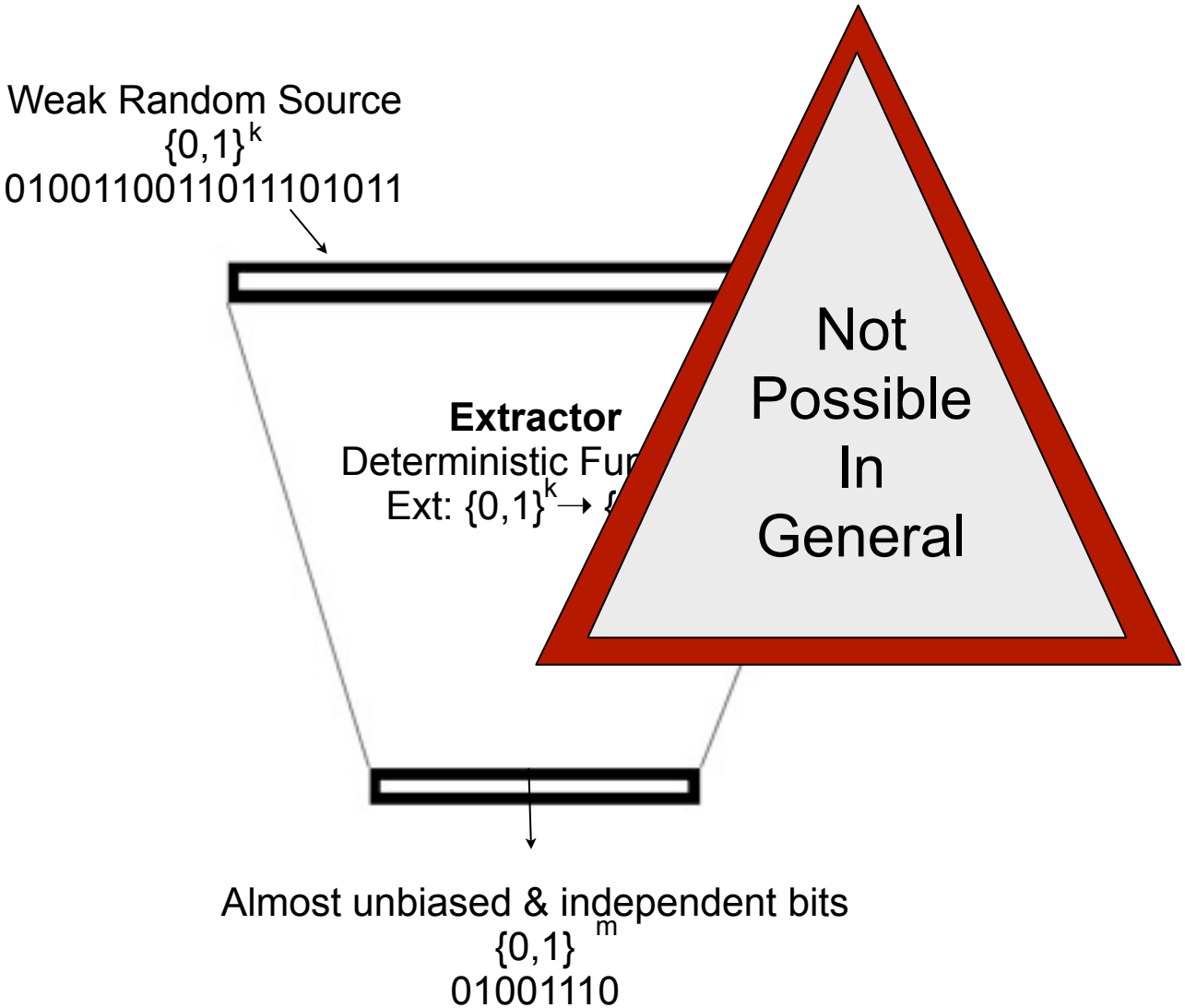
4. Randomness In A Distributed Environment

Extractors



4. Randomness In A Distributed Environment

Extractors



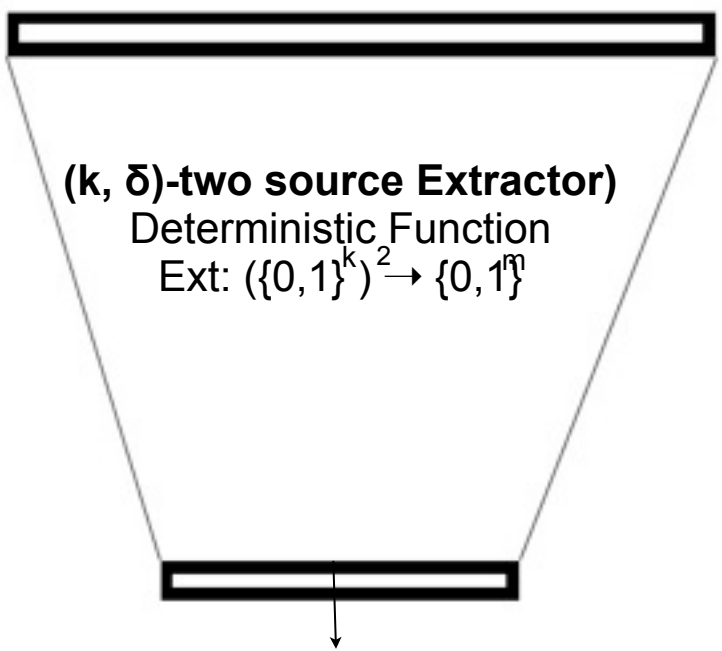
4. Randomness In A Distributed Environment

2 Source Extractors

Weak Random Source
0100110011011101011

←independent→

Weak Random Source
0100111011010011011



Almost unbiased & independent bits
$\{0,1\}^m$
01001110

4. Randomness In A Distributed Environment

A Very Simple Extractor

4. Randomness In A Distributed Environment

A Very Simple Extractor

0 | 1

4. Randomness In A Distributed Environment

A Very Simple Extractor

0	1
0	0

4. Randomness In A Distributed Environment

A Very Simple Extractor

0	1
0	0
1	1

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0	1
0	0
1	1
1	0

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0	1
0	0
1	1
1	0
0	0

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0	1
0	0
1	1
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0	1
0	0
1	1
1	0
0	0
1	0
0	1

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A Very Simple Extractor

0	1	0
0	0	
1	1	
1	0	1
0	0	
1	0	1
0	1	0

4. Randomness In A Distributed Environment

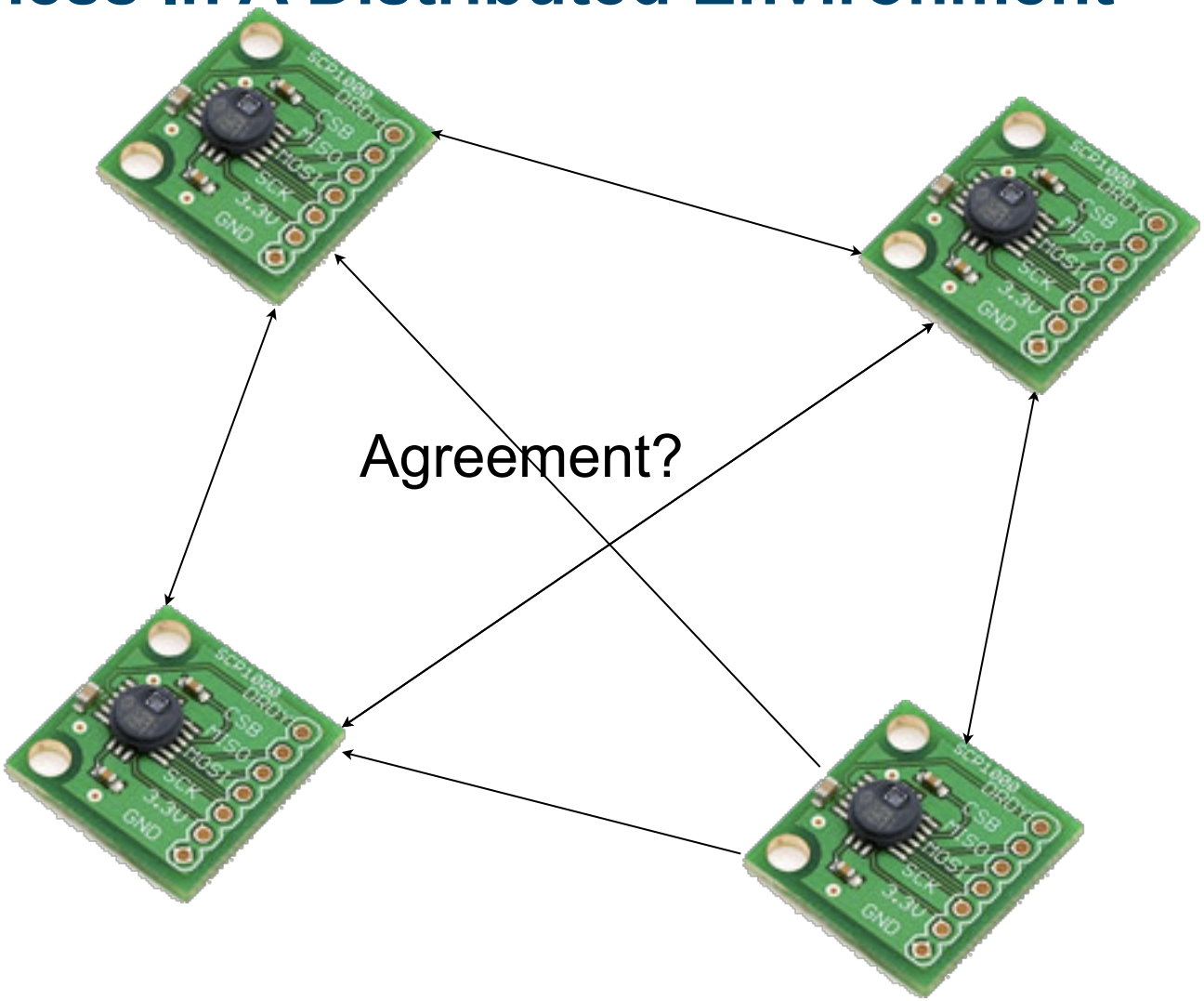
A Very Simple Extractor

0	1	0
0	0	
1	1	
1	0	1
0	0	
1	0	1
0	1	0

Not a (k, δ)-two source Extractor!
(do you see why?)

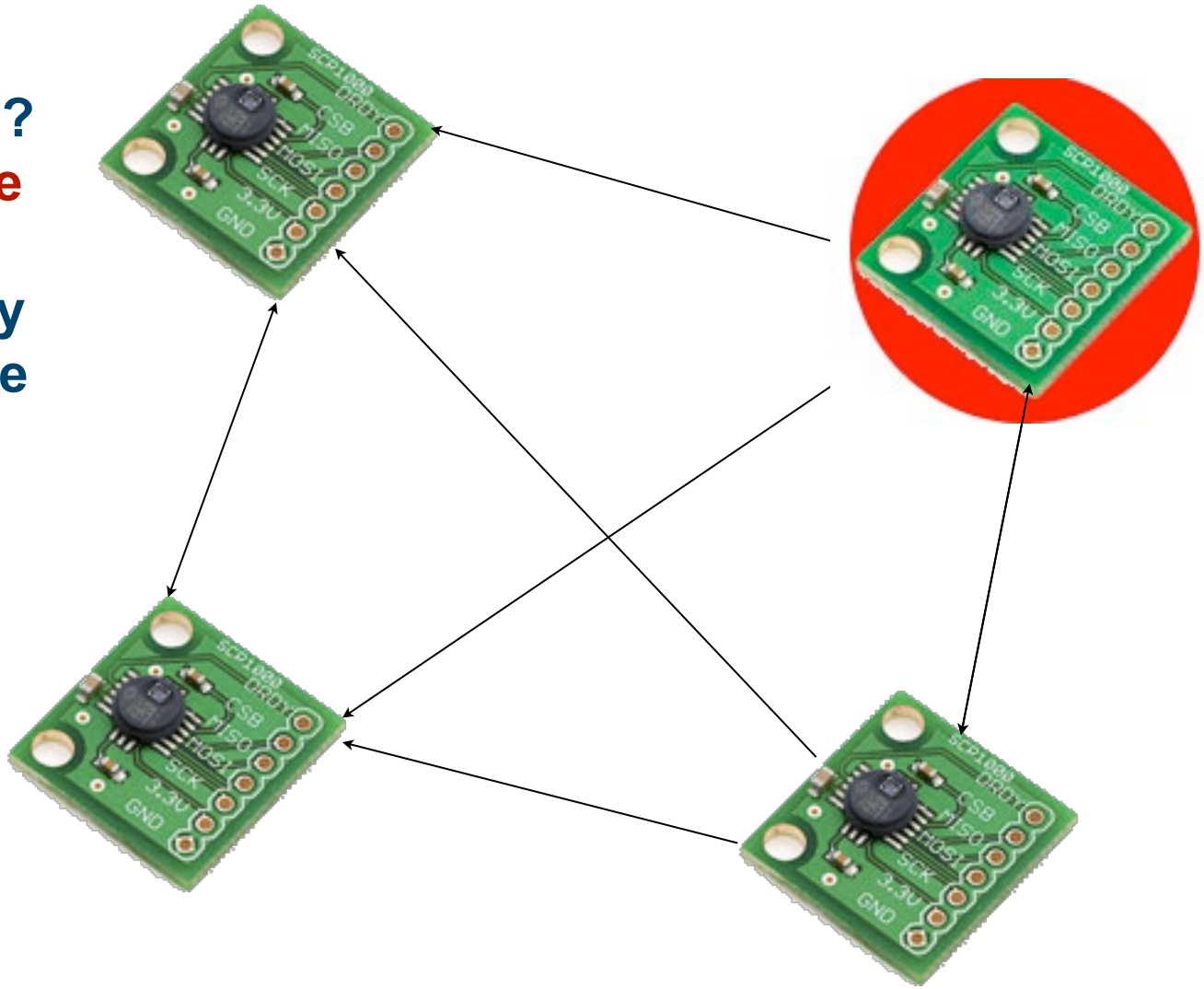
4. Randomness In A Distributed Environment

Is this enough?



4. Randomness In A Distributed Environment

Is this enough?
No, as we have faulty players!
What if they try to "poison" the randomness?



4. Randomness In A Distributed Environment

Immune Extractors

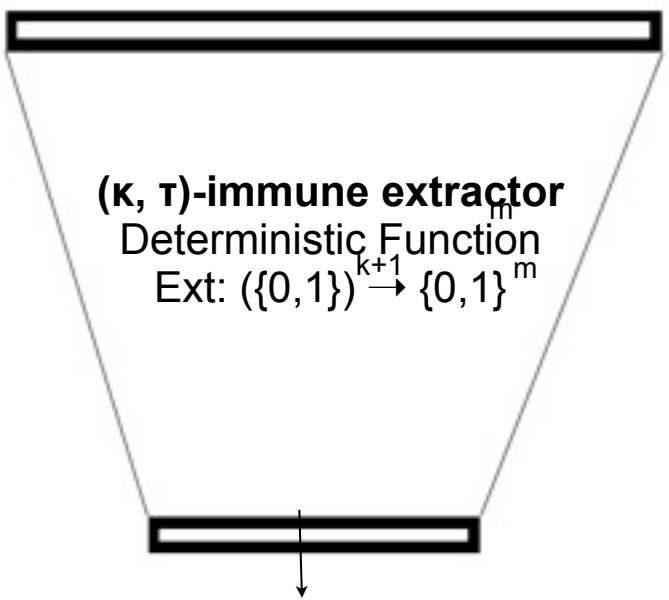
- **Multi-source extractors whose output is random even if an arbitrary subset of the input sources don't send a weak random string (e.g. faulty players) but all sources are independent**

4. Randomness In A Distributed Environment

(κ, τ) Immune Extractors, as a picture

κ + 1 block sources

(k, δ) source (k, δ) source (k, δ) source (k, δ) source



Almost unbiased & independent bits

$\{0,1\}^m$

01001110

4. Randomness In A Distributed Environment

I-Ext: A(t, t-1)-immune extractor

- Let Ext be any (k, δ) two source extractor
- Let X_1^2, X_1^3, ... denote t distinct blocks from the source X_1
- Let X_2, X_3, ... be one block each from the other t sources
- I-Ext($\{X_1^i\}_{i=2}^{t+1}$, X_2, ..., X_{t+1}) = $\oplus_{i=2}^{t+1}$ Ext(X_1^i, X_i)

4. Randomness In A Distributed Environment

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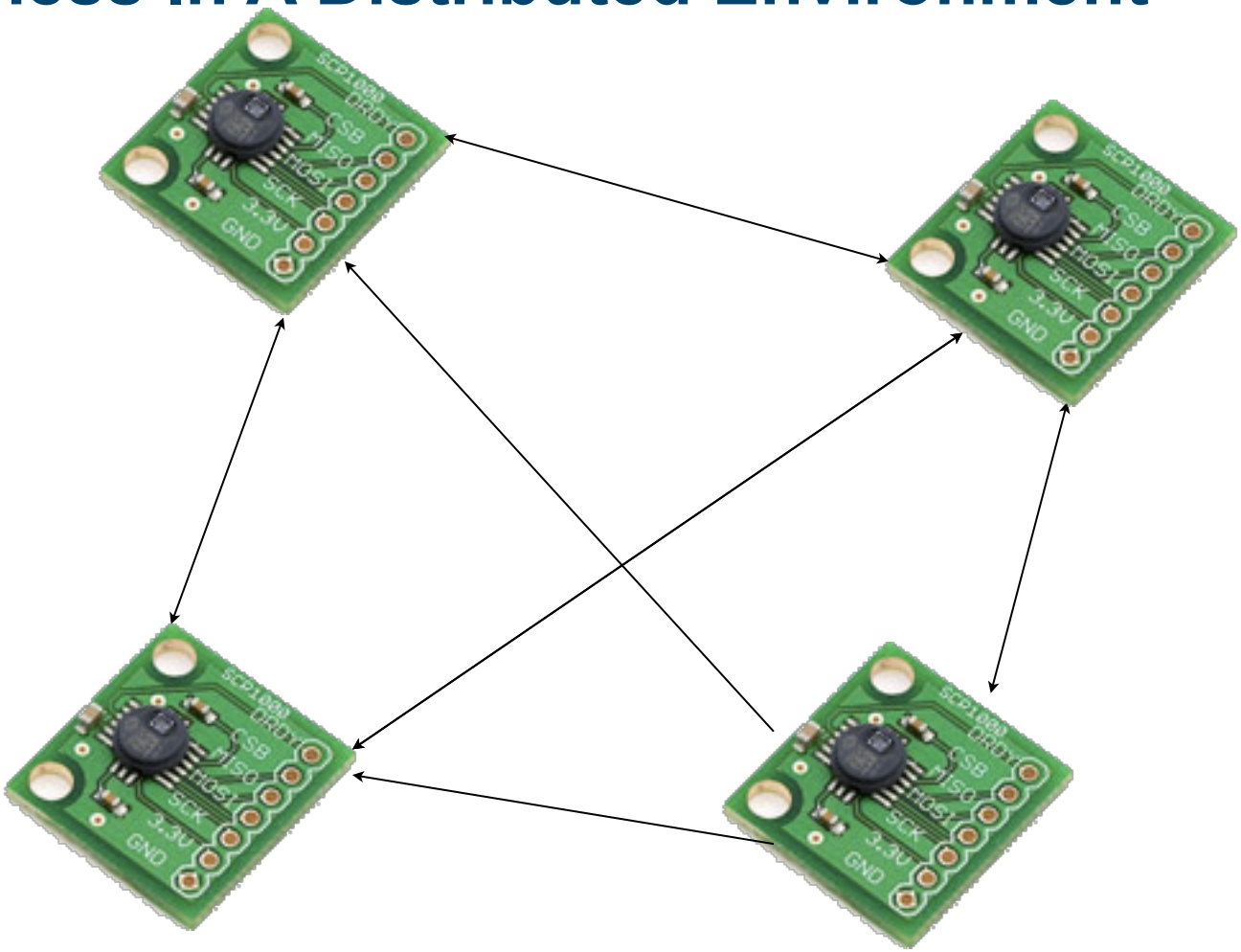
- Theorem: I-Ext is a (t, t-1) immune extractor!

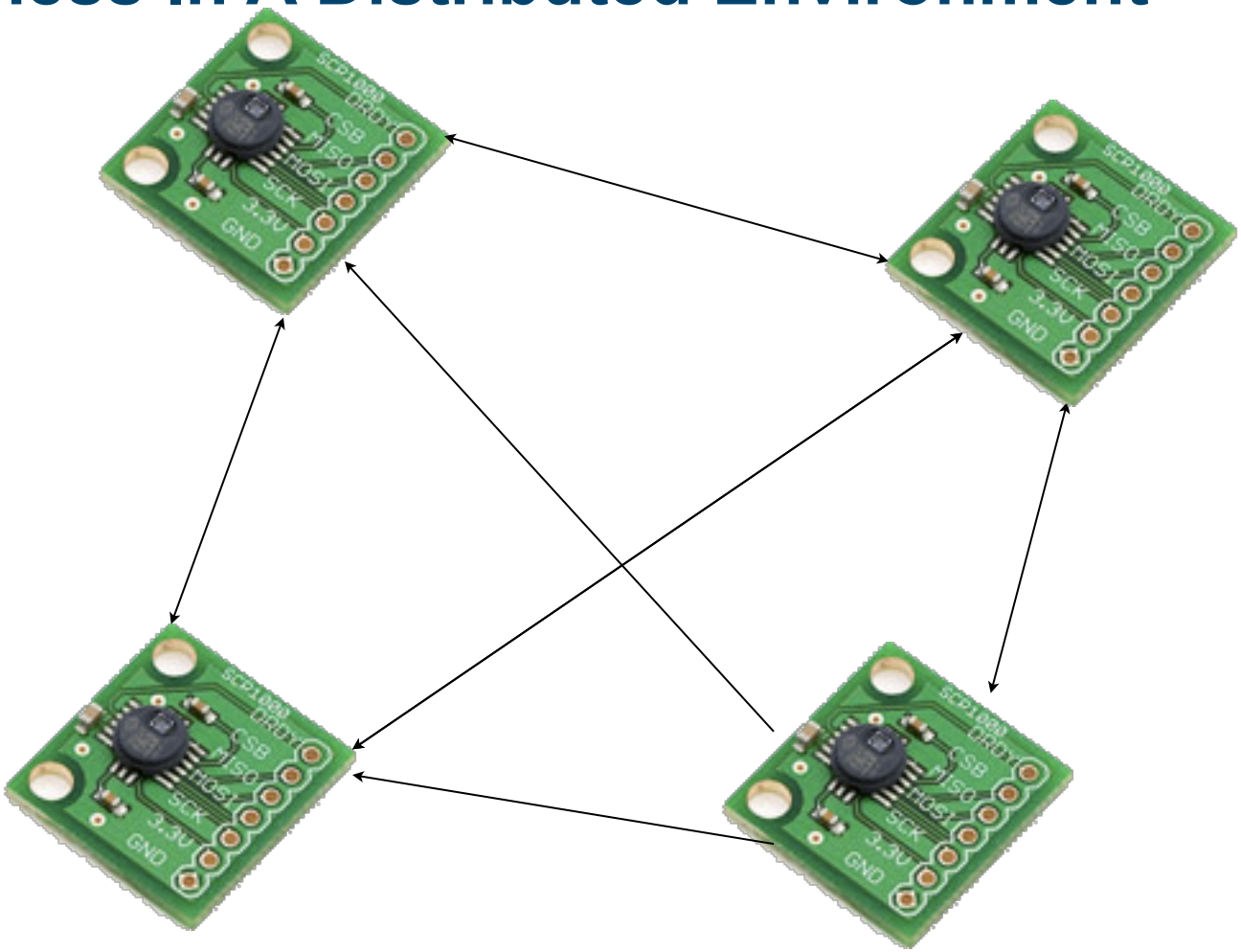
i

4. Randomness In A Distributed Environment



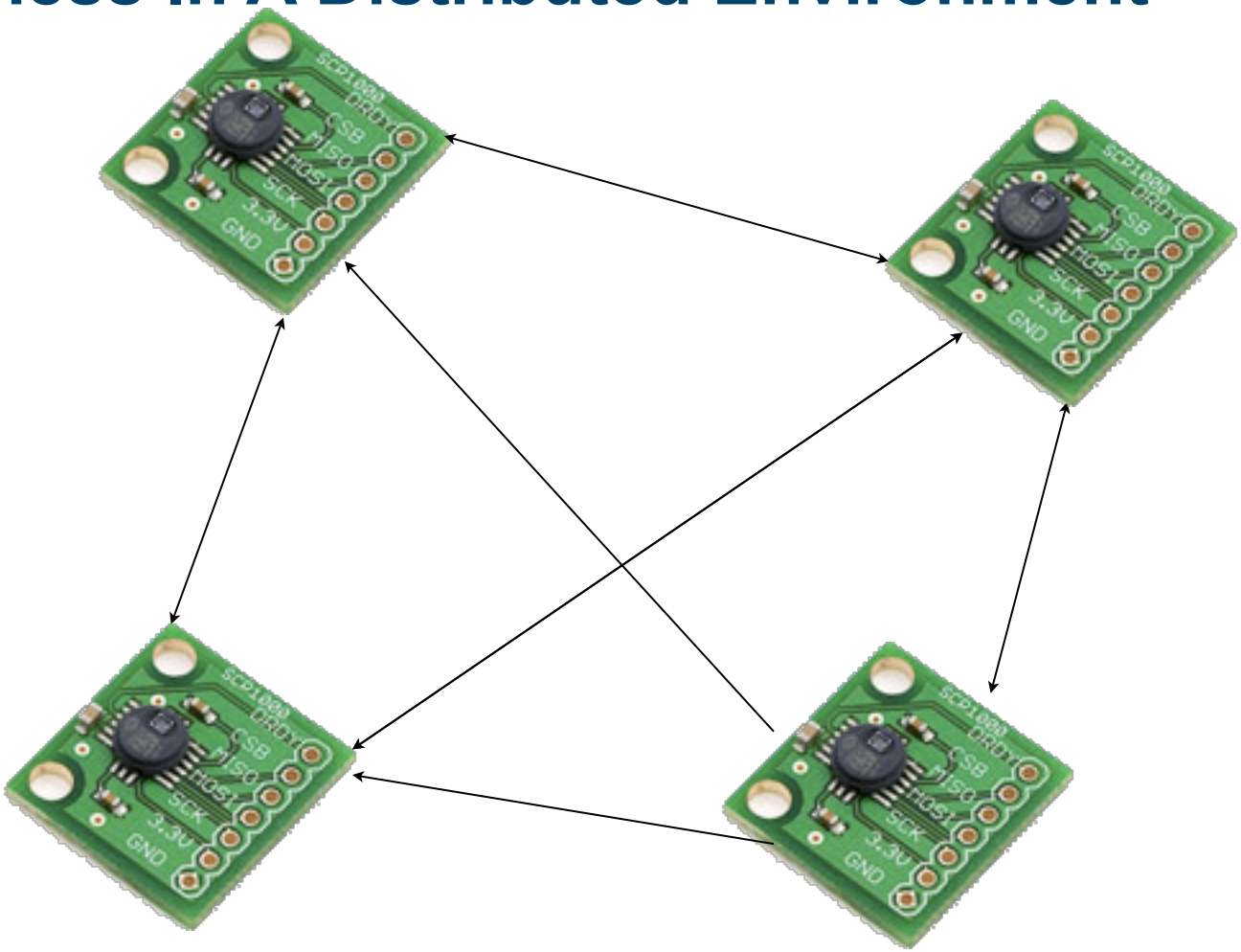
... each player sends each other player a string from its weak random source ...

4. Randomness In A Distributed Environment



... each player can use I-Ext to generate a string very close to U_m ..

4. Randomness In A Distributed Environment



... every player has an almost random string ...

5. Conclusion

The paper mainly showed how it is possible to for each player of a network to extract a almost unbiased, uniform random string by sharing their (mutually independent) random sources, even in presence of faulty or even malicious parties.