Category-based routing in social networks

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Overview

- Small-world phenomenon
- Definitions and routing algorithm
- Building categories
- Summary

Part 1

Small-world phenomenon

Small world phenomenon

- The phenomenon of elements being perceived to be very far away but are only at a distance of few hops

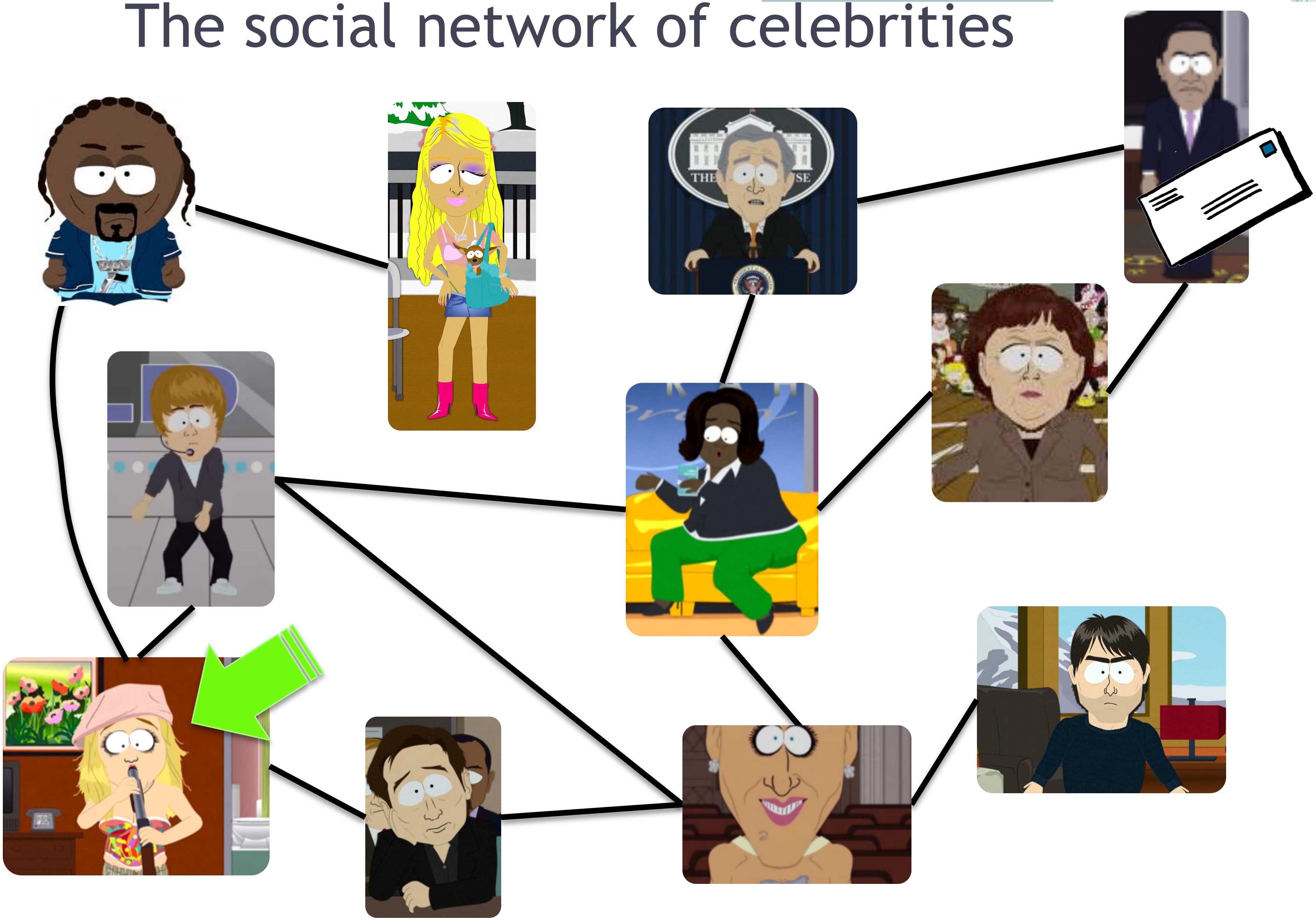
- The experience of meeting a complete stranger and finding out you share a mutual friend

  - "It's a small world" … we say

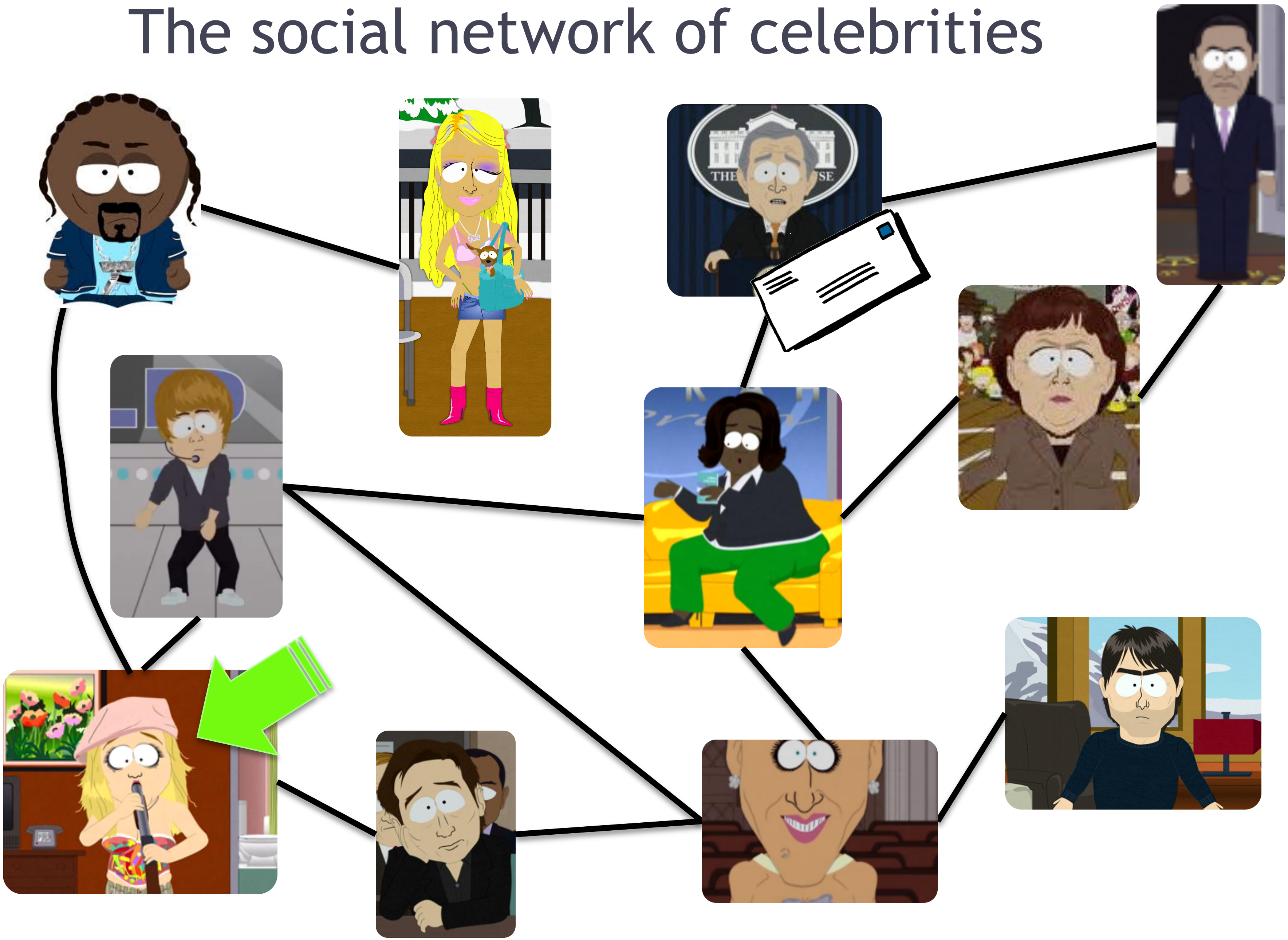
The notion of the small world phenomenon

- Milgram's experiment form the 1960's
  - Given: Social network
  - Idea: How many jumps are needed to reach a random person?
  - Starters: Random people from Nebraska
  - Target: Person, who lived in Massachusetts and worked in Boston
  - Known: Basic information about the target
  - Rule: Send to a people known on first name basis
- Routing using only local information and simple facts

The social network of celebrities



The social network of celebrities



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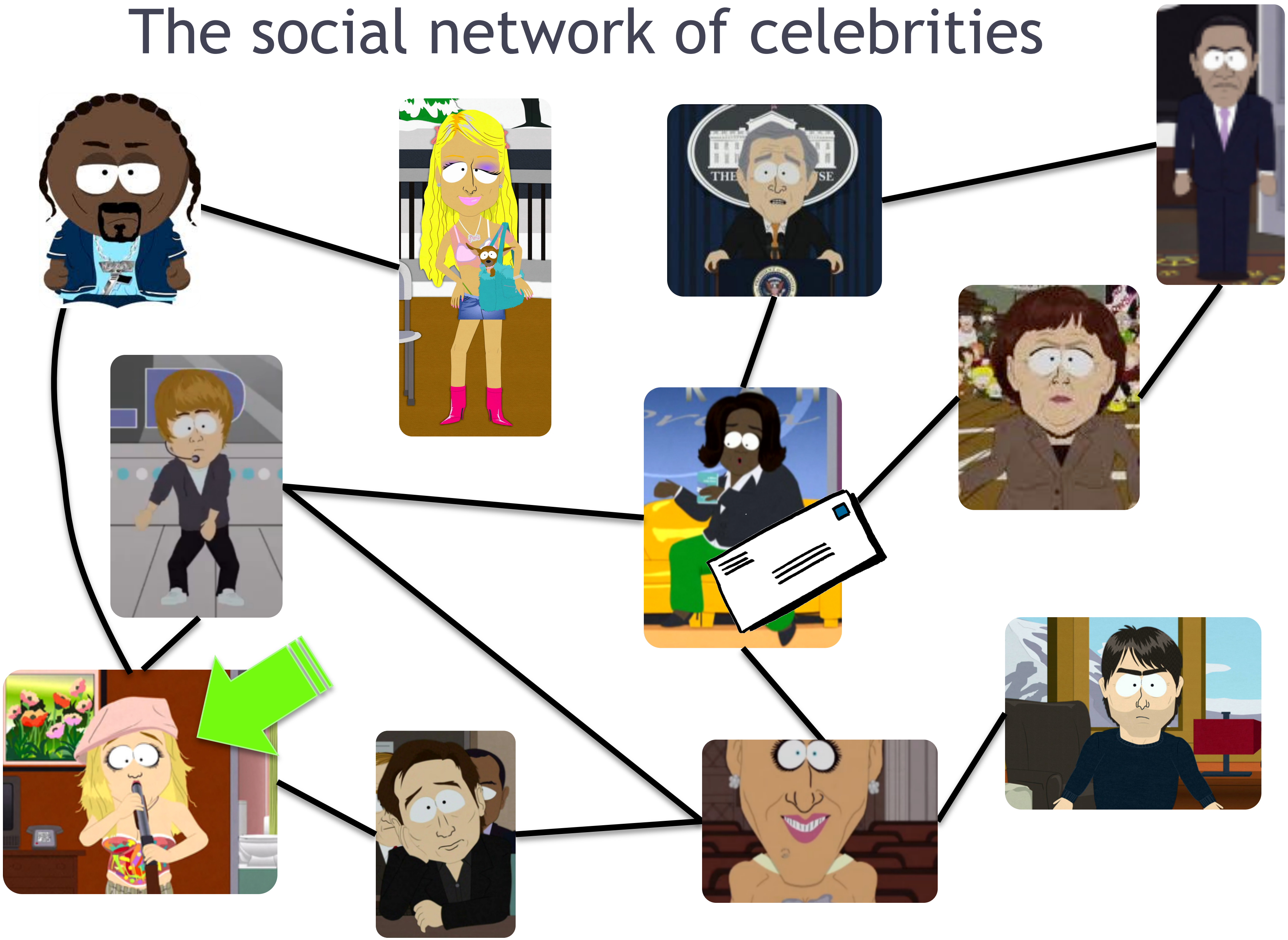

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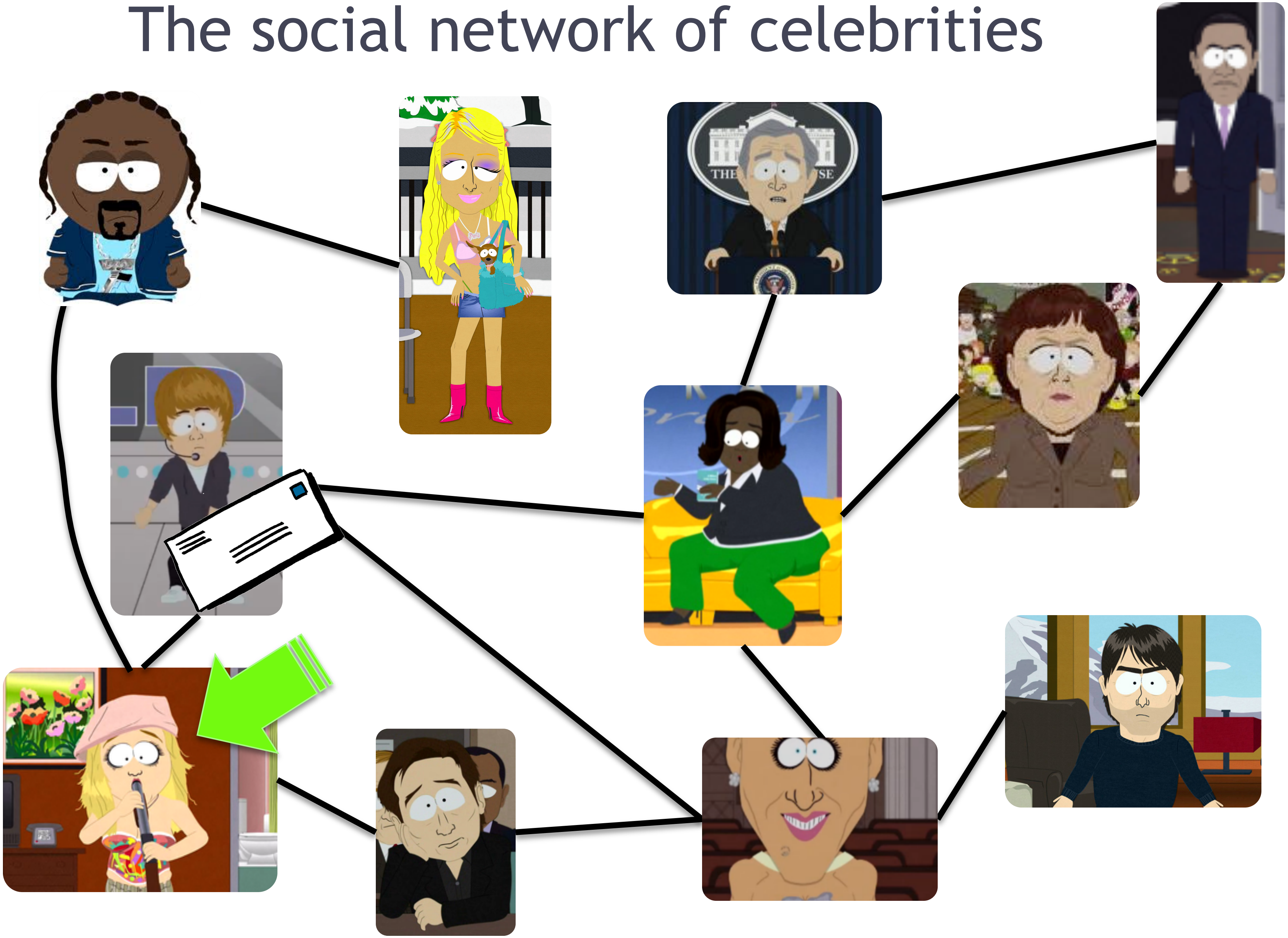

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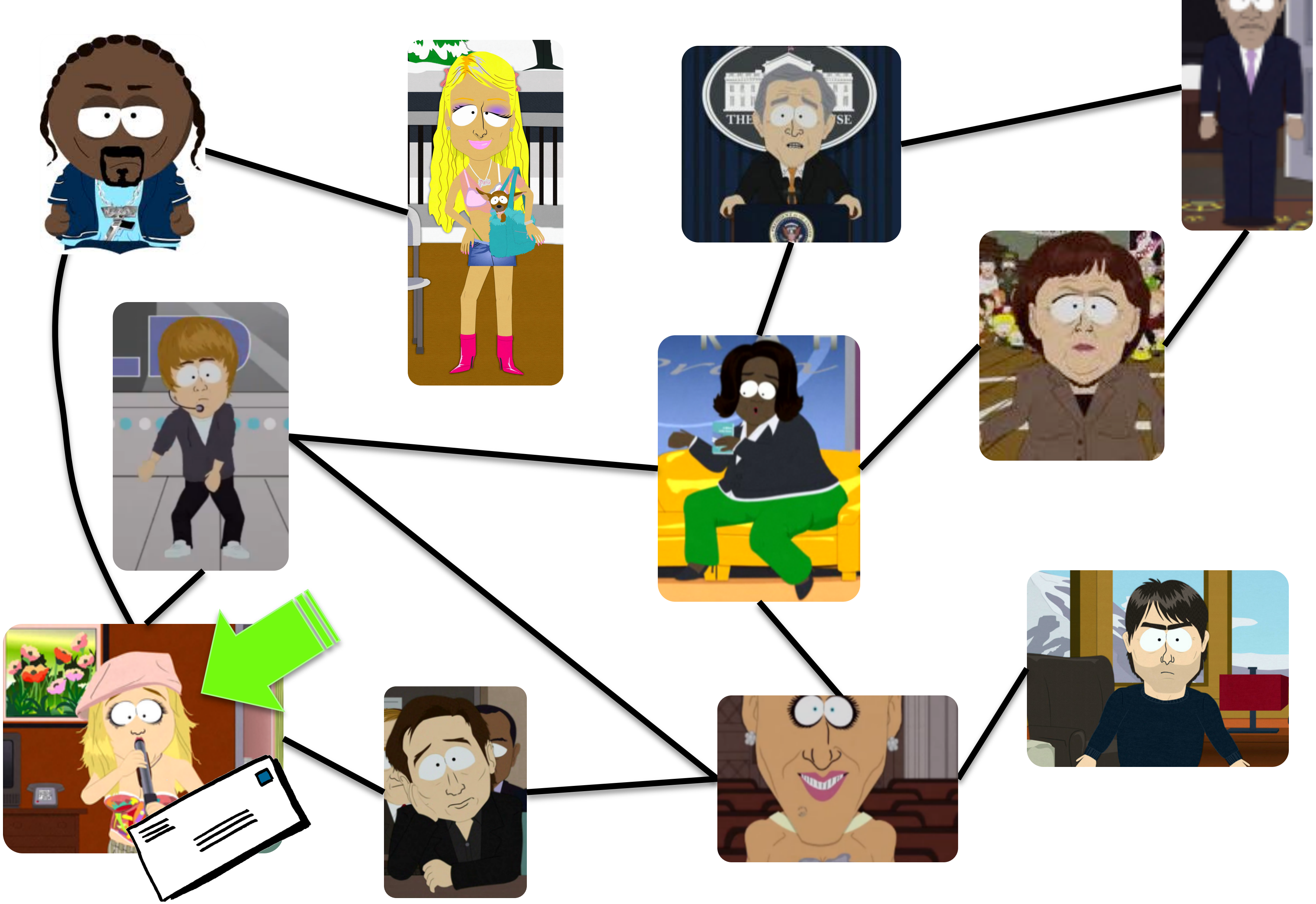

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- Routing using only local information and simple facts
- Routing works!

  … and uses always very short paths

Results from Milgram’s experiment

- Chain: between 2 and 10 intermediate acquaintances
- Median: 5 intermediates
- Conclusion: Any person appeared to be reachable in just 6 jumps

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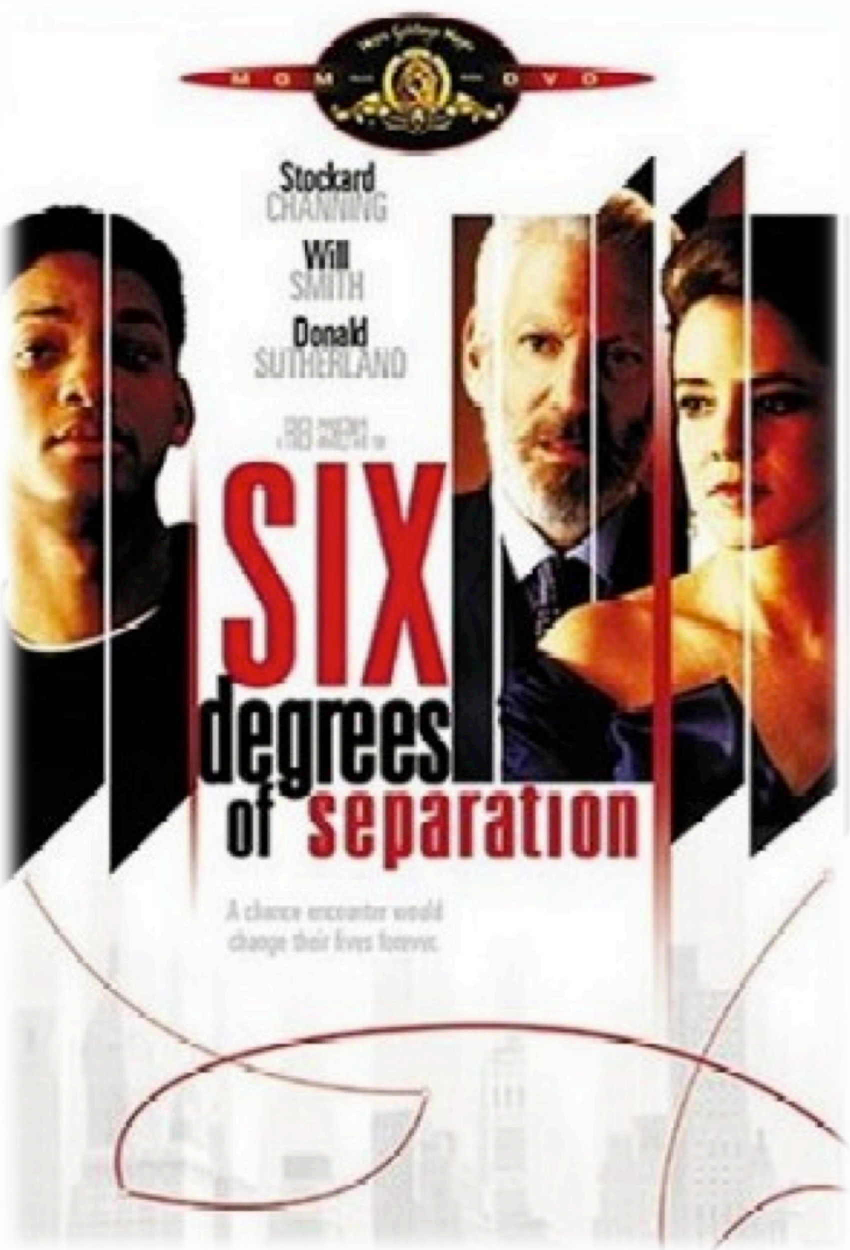
→ "Six degrees of separation"


MGM DVD
Stockard CHANNING
Will SMITH
Donald SUTHERLAND
SIX degrees of separation
A chance encounter would change their lives forever.

Six Degrees of Kevin Bacon

The Oracle of Bacon:

Six Degrees of Kevin Bacon

The Oracle of Bacon

Importance

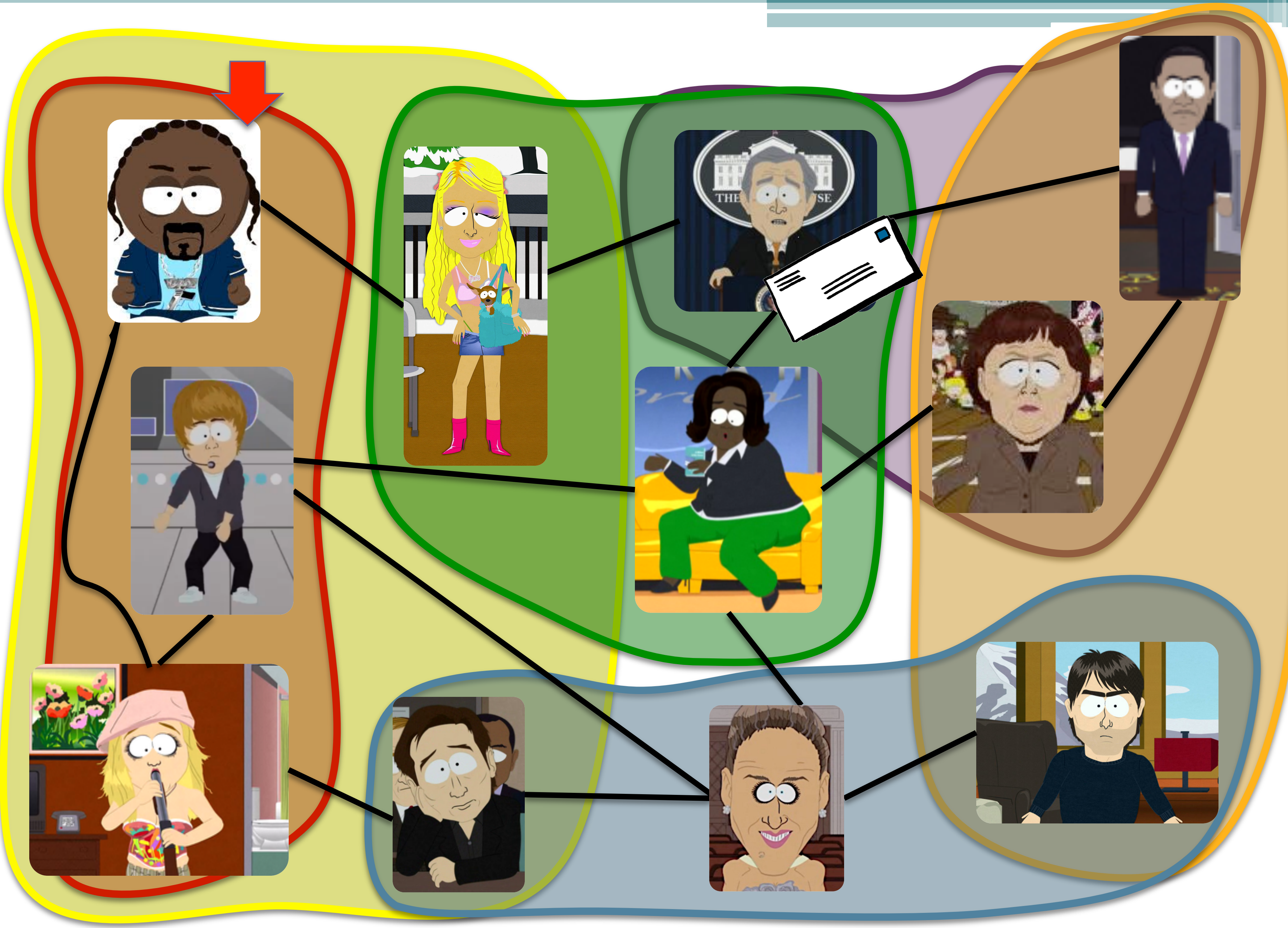
- Indicative of the underlying structure of modern social networks

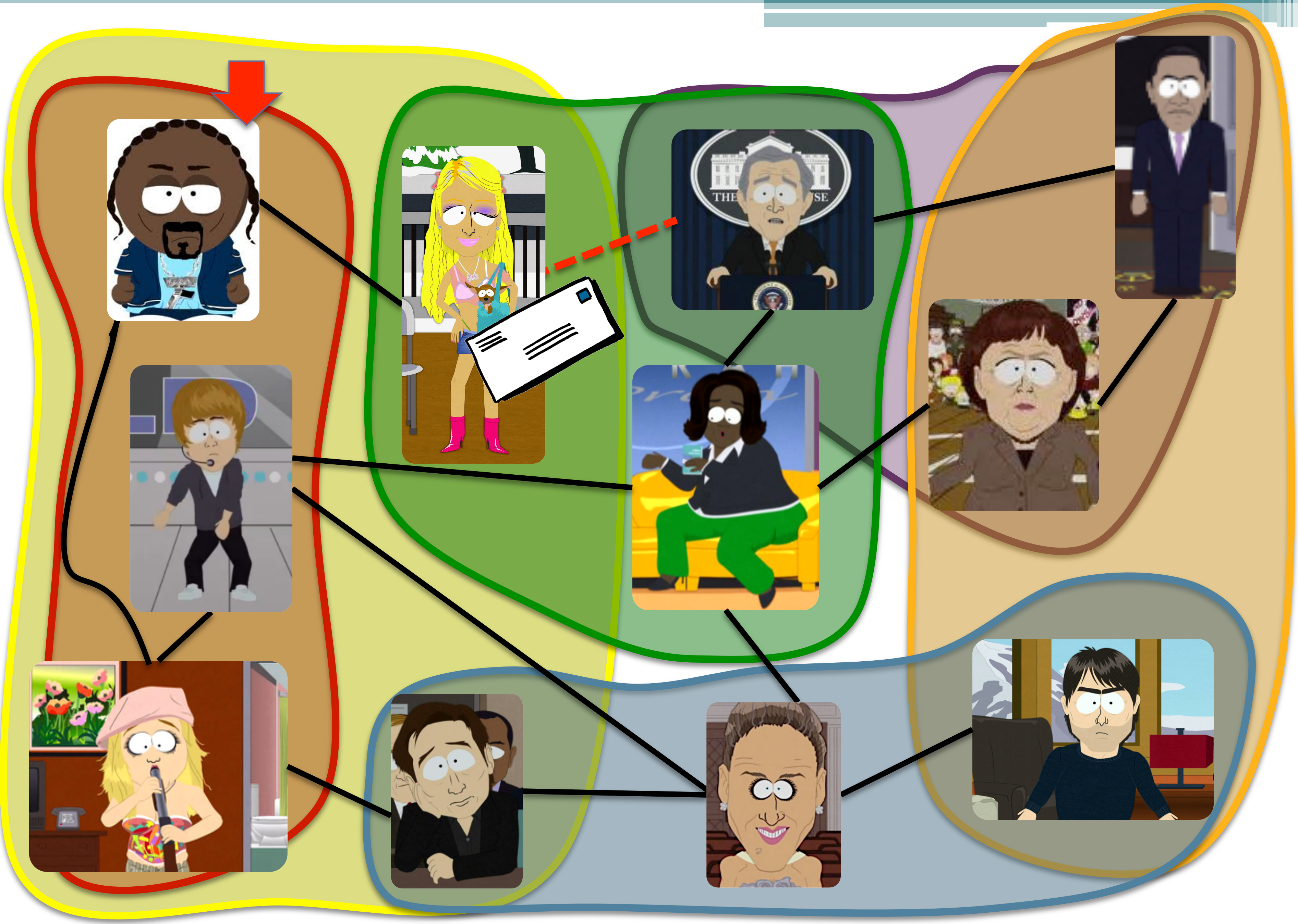
- In what other context:
  - infectious disease spreading
  - computer virus transmission

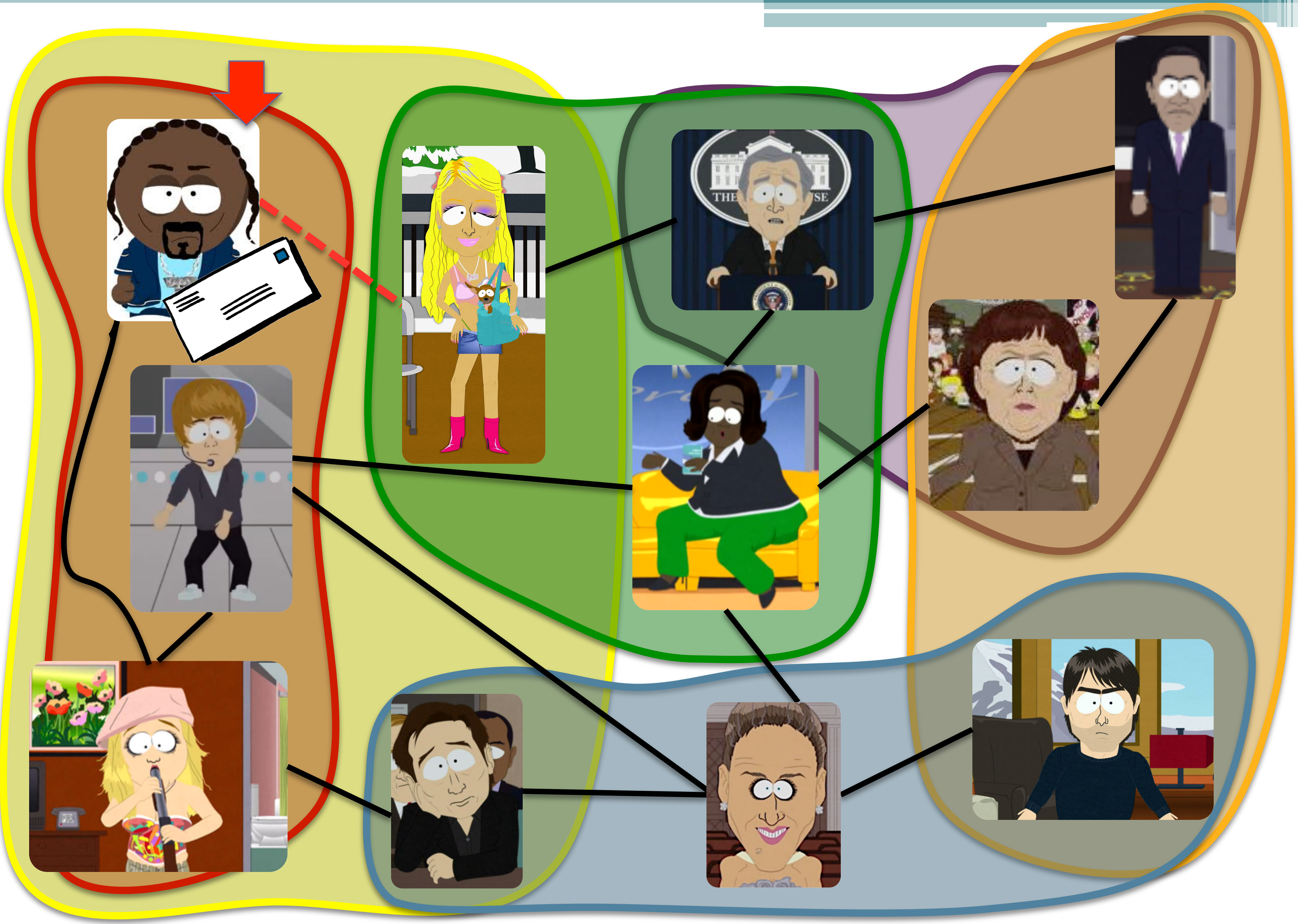
Observation from Milgram's experiment

- Decisions people take for selecting routes are overwhelmingly categorical in nature
- Categories are based on different factors like:
  - occupation, location, ethnicity

Consider a set of categories
… in the scenario of a social network of celebrities





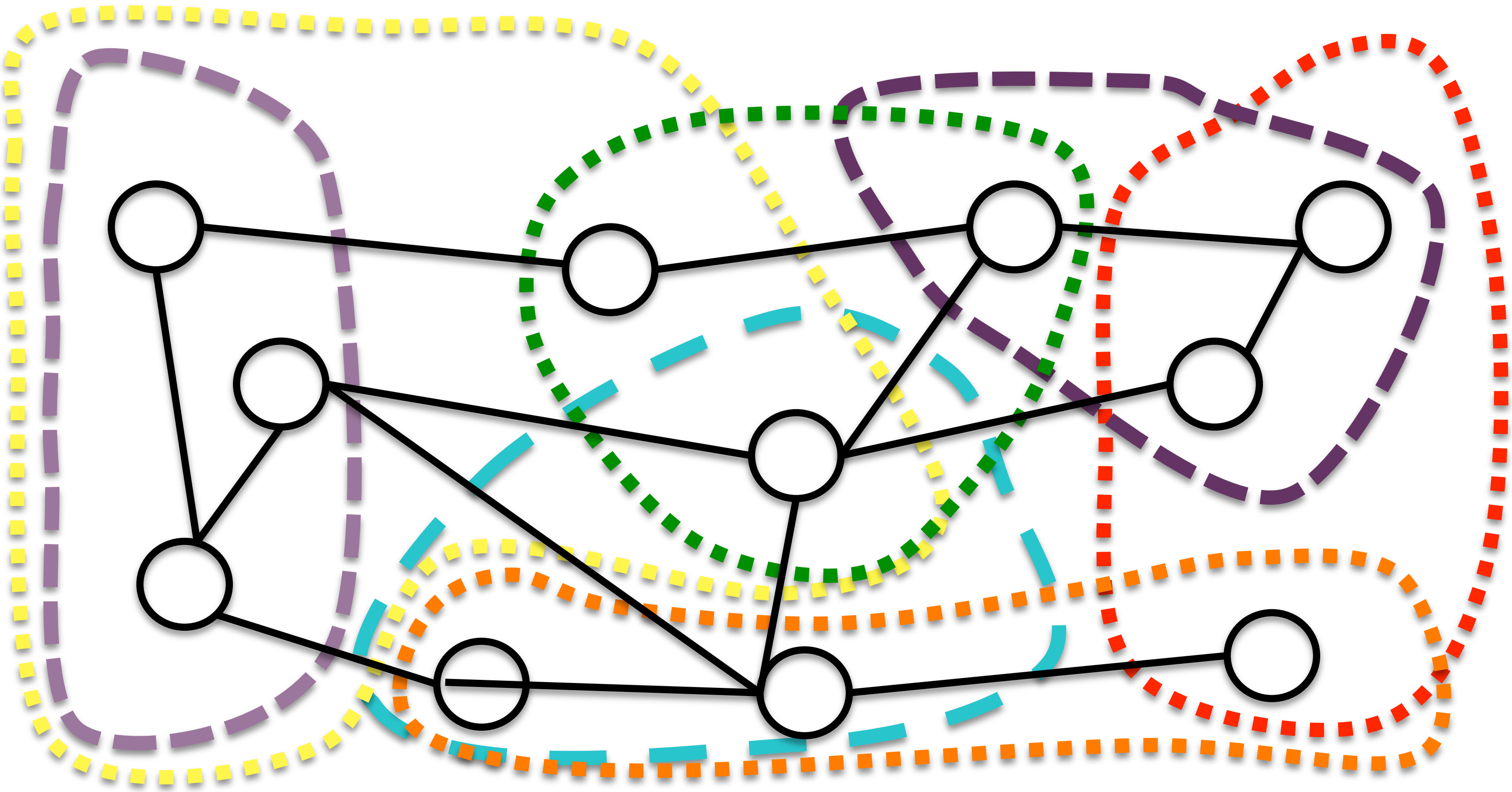


Part 2

Definitions and routing algorithm

Definitions:

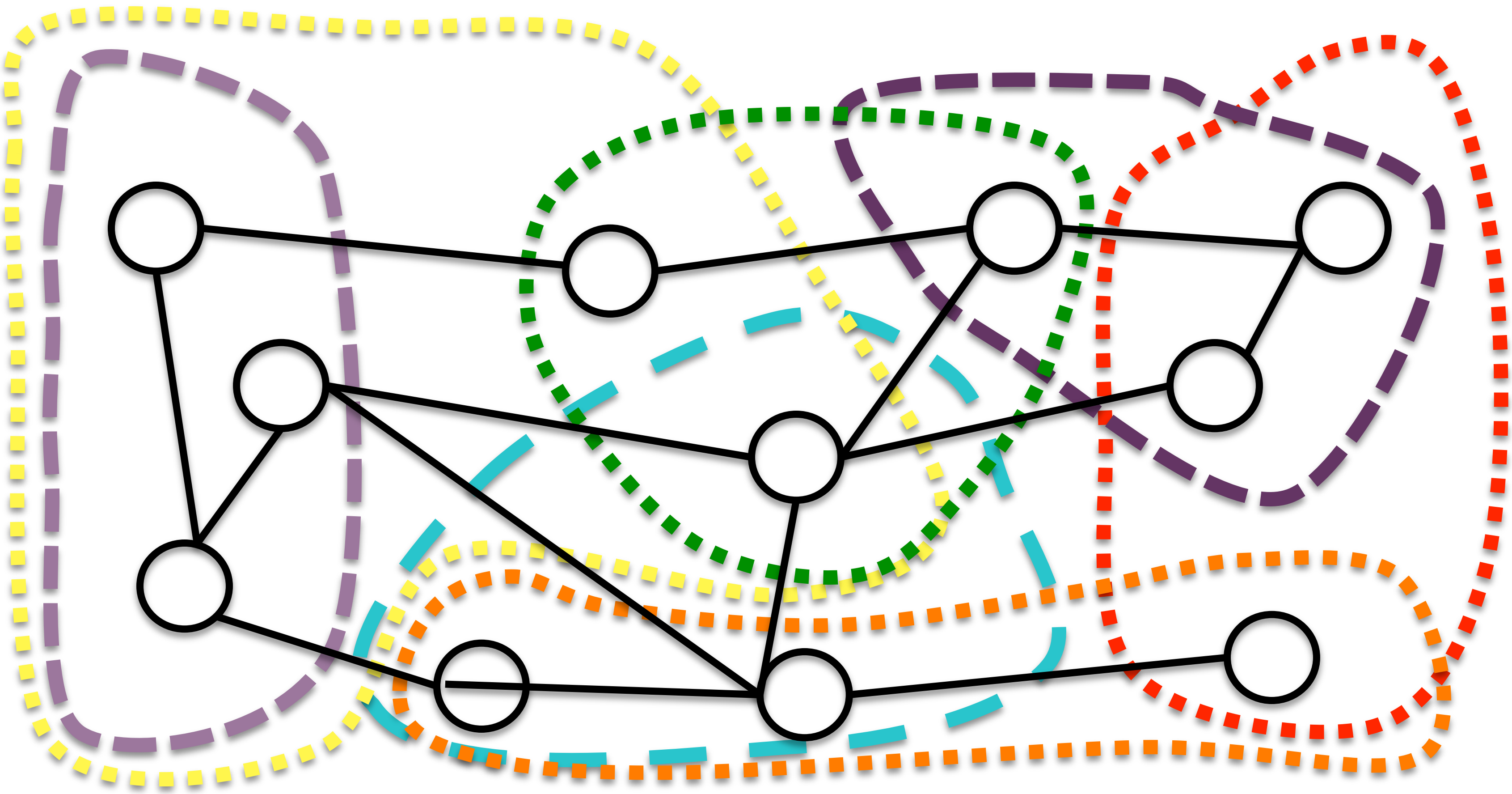
- Collection of categories: $S \subset 2^U$



Definitions

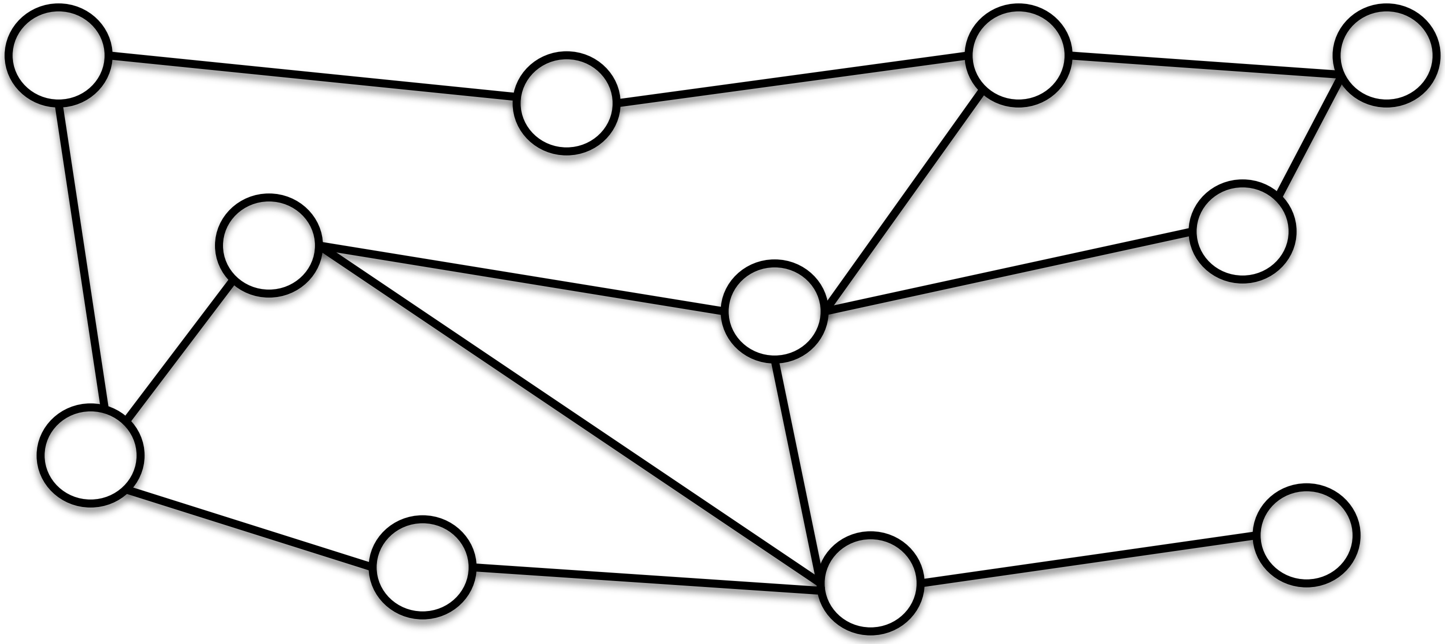
- Membership dimension: $\text{memdim}(S) = \max_{u \in U} |\text{cat}(u)|$

where $\text{cat}(u) = \{C \in S \mid u \in C\}$ is the set of categories of a node u



Definitions

- Maximum length of any shortest path:
  $diam(G)=max_{s,t \in U} sp(s,t)$



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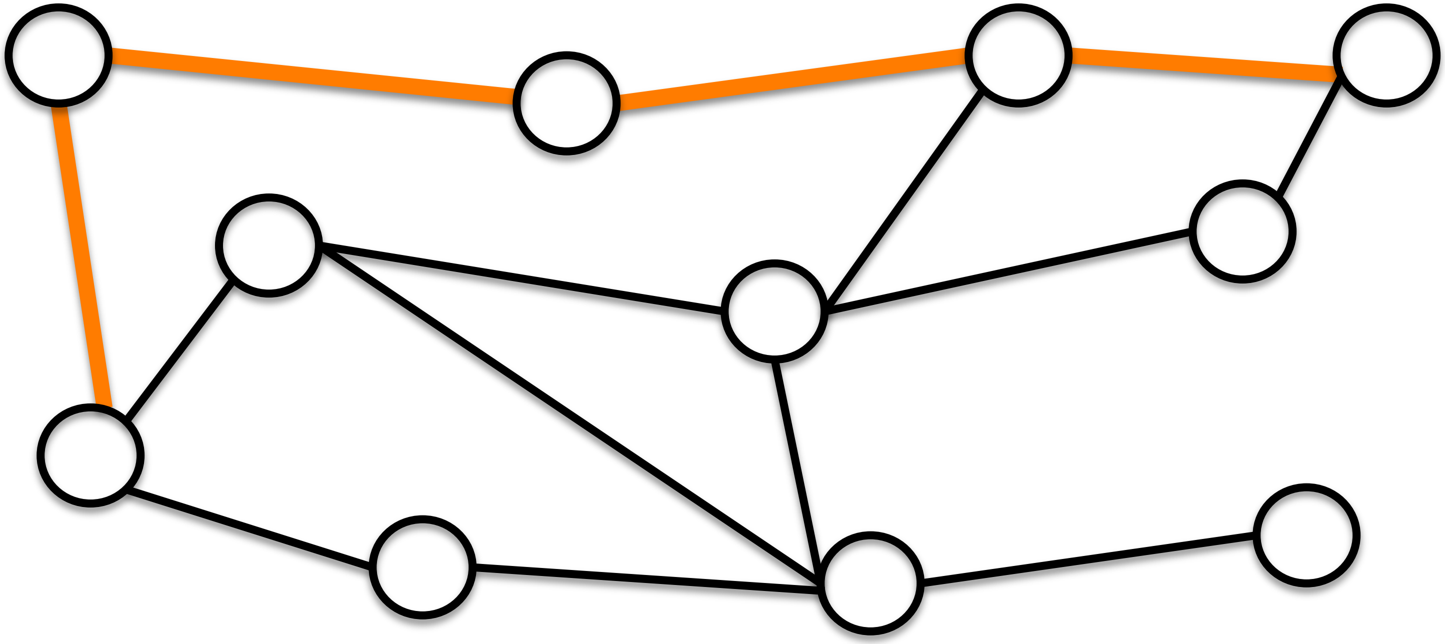
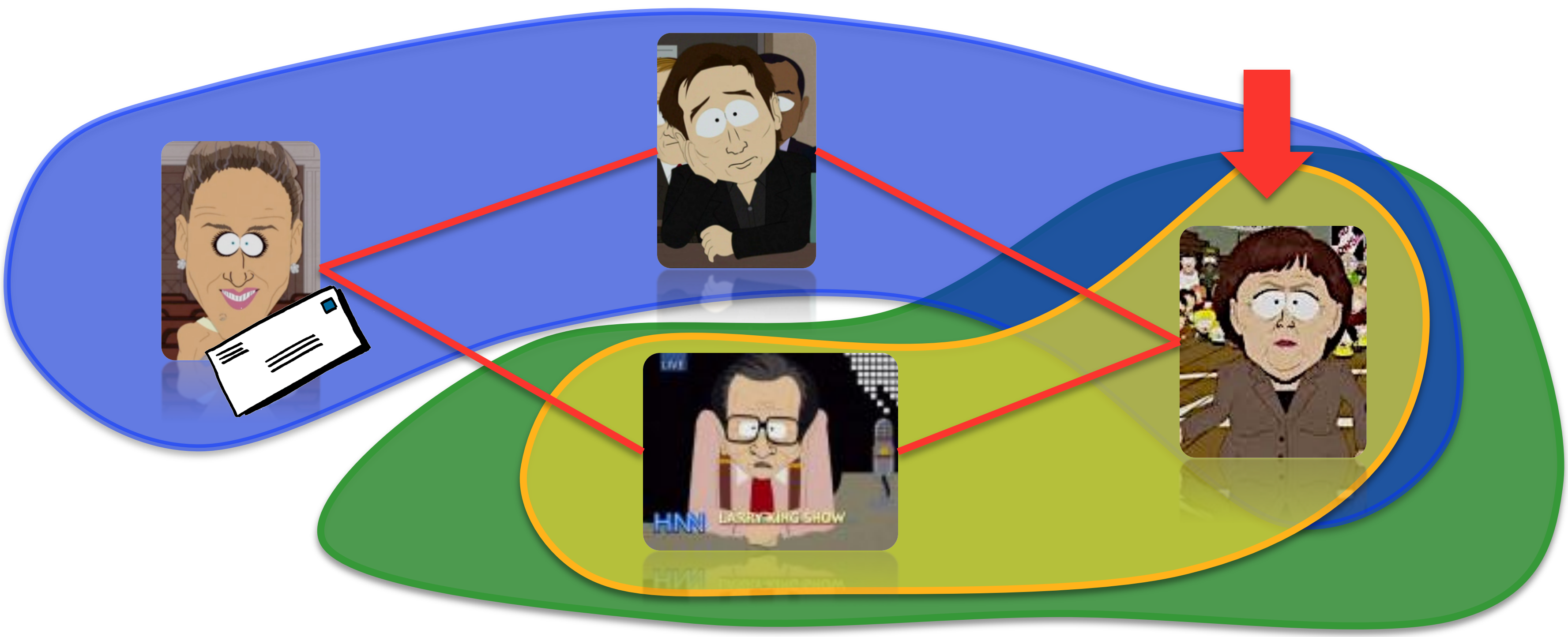


Greedy routing algorithm

- The category-based distance function: $d(s,t) = |cat(t) \setminus cat(s)|$
- Sending from u to w: forward to a neighbor v that is closer to w than u: $d(v,w) < d(u,w)$



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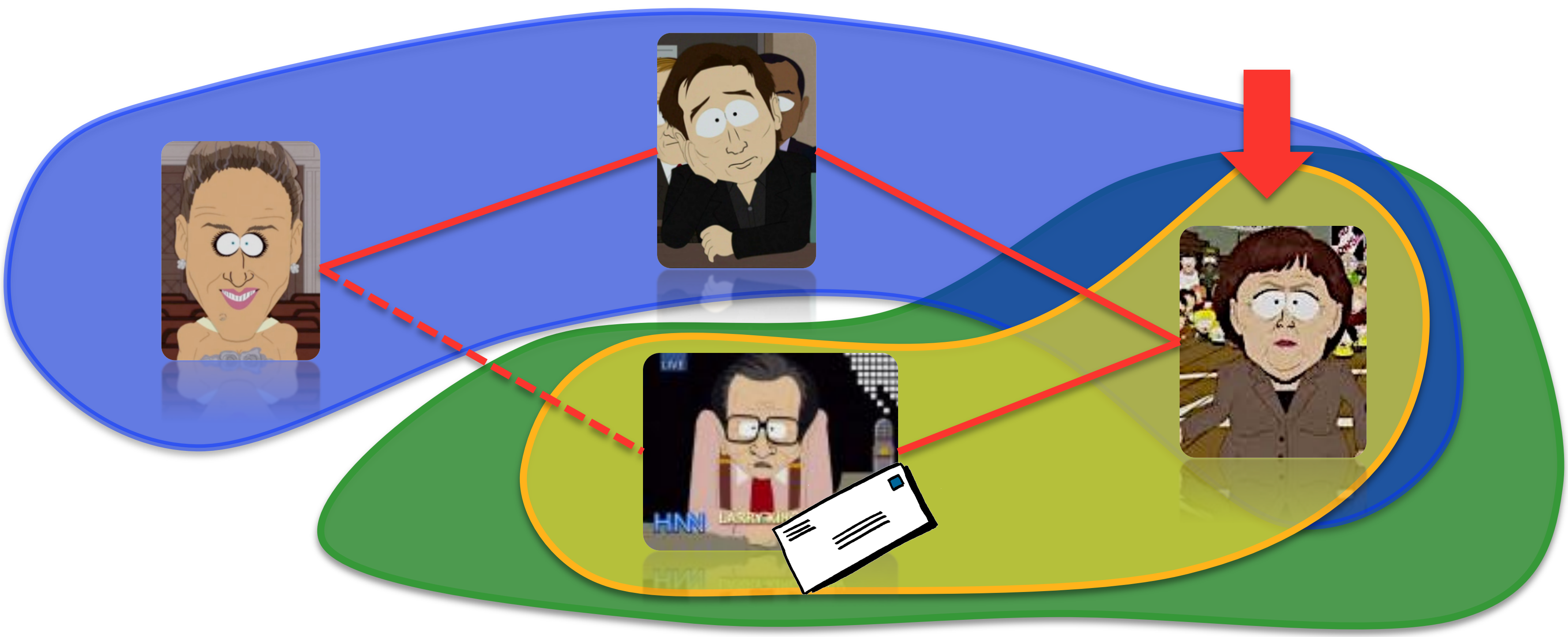

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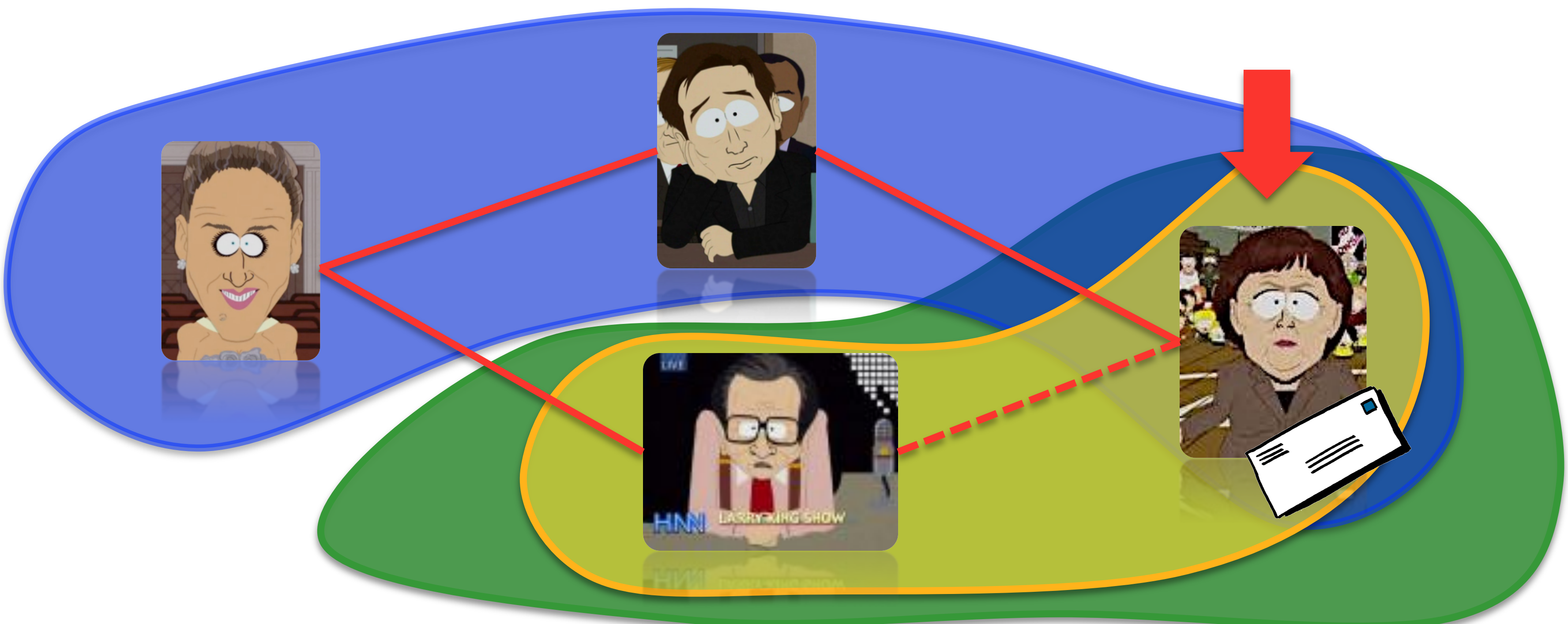


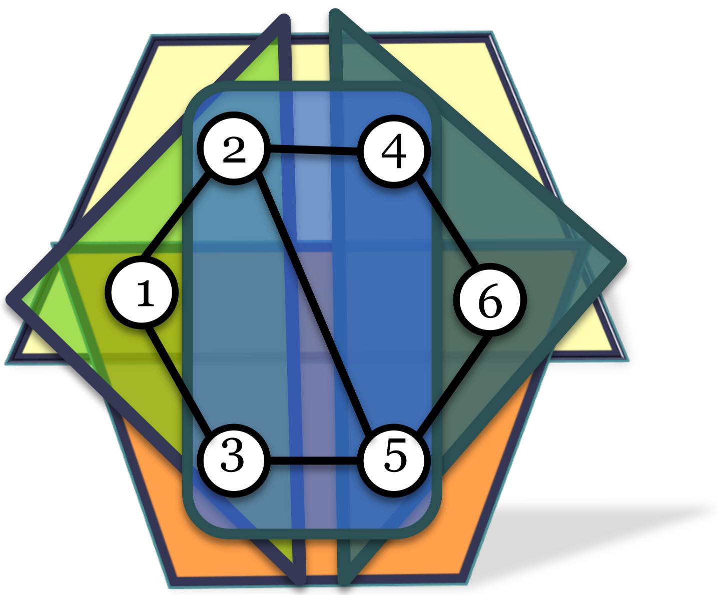
Properties of successful routing

- Internally connected:

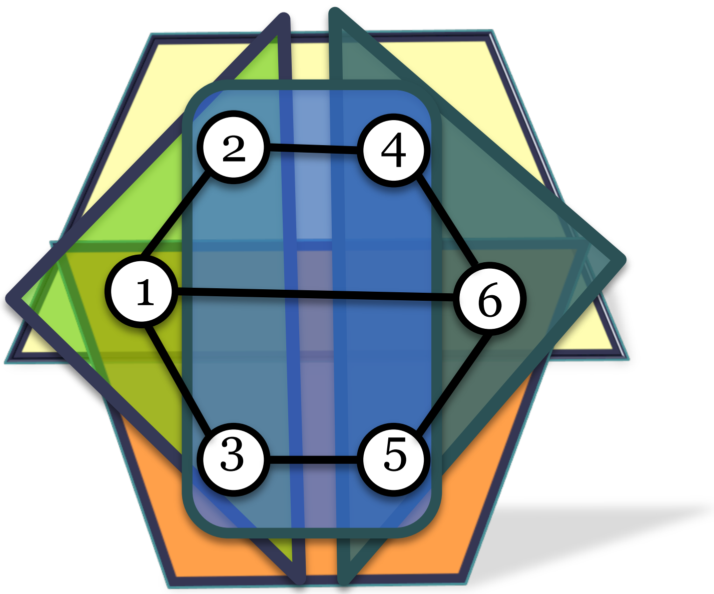
  (G, S) is internally connected, if for each C $\in$ S, G restricted to C is internally connected

- Shattered: A pair (G, S) is shattered if, for all $s, t \in U, s \neq t$, there is a neighbor u of s and a set $C \in S$ such that C contains u and t, but not s.

Examples



Internally connected
&
Not shattered



Shattered
&
Not internally connected

Shattered

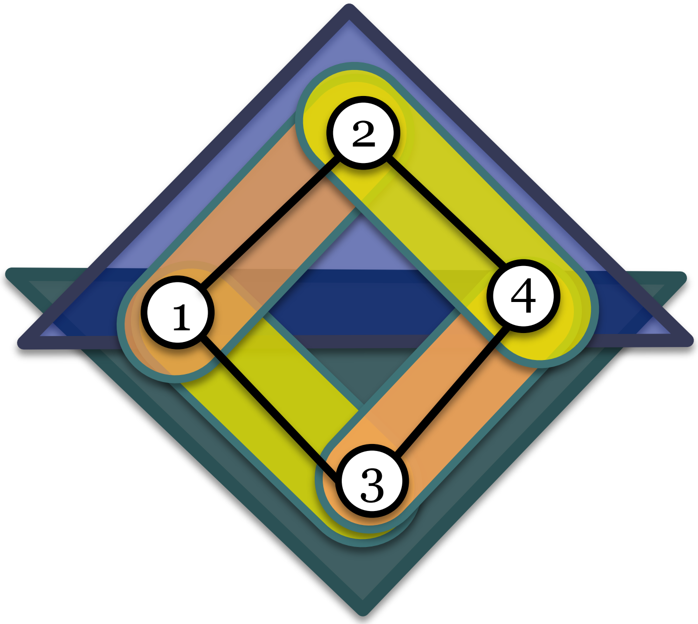
- In order for someone to advance a letter to a target, there must be an acquaintance that shares additional interests with the target.

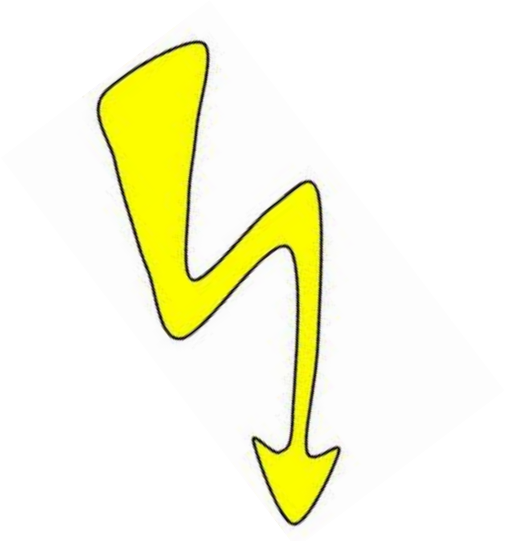
- **Lemma 1**: If (G, S) is not shattered, Routing fails.

What about routing in trees?

Lemma 2:
If G is a tree, and (G, S) is internally connected and shattered, then Routing is guaranteed to work.

- Not enough for arbitrary connected graphs





Summary of the definitions

- memdim(S)= $\max_{u \in U} |cat(u)|$
- $diam(G) = \max_{s,t \in U} sp(s,t)$
- Routing: forward to a neighbor v that: $d(v,w) < d(u,w)$
- Internally connected
- Shattered

Part 3

Building categories

Lower and upper bounds

If G and S are a graph and a category system such that Routing works:

- memdim(S) ≥ diam(G)
- $\text{memdim(S)} = O((\text{diam}(G) + \log n)^2)$

Lower bound of the cognitive load

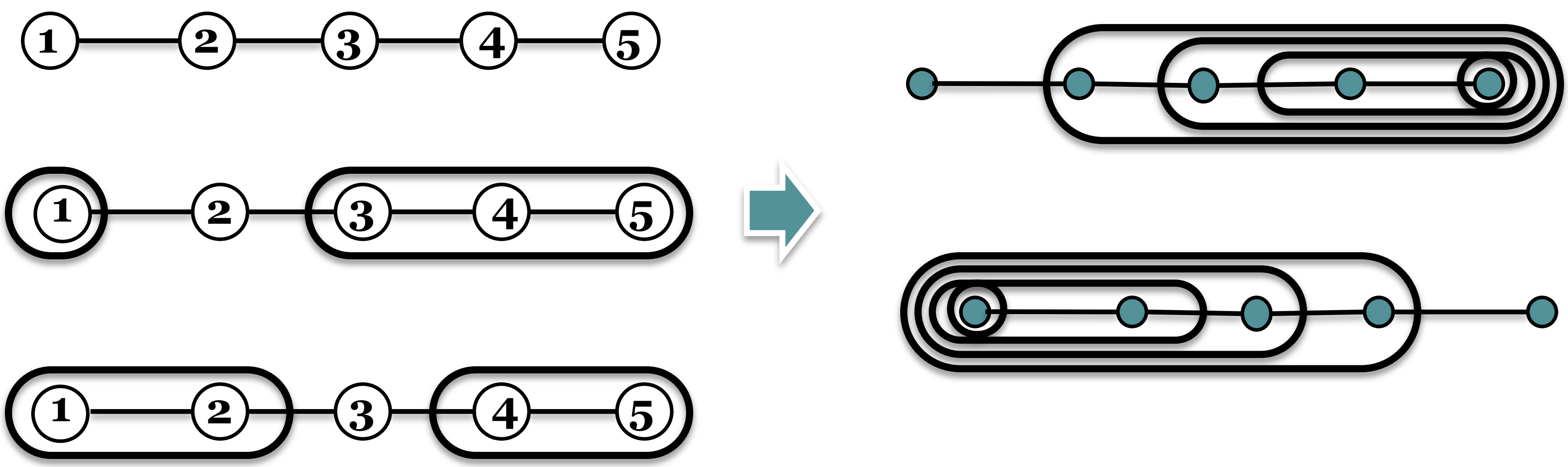
- Lemma 3:

If (G,S) be a graph and a category system, respectively, such that Routing works for G and S. Then $\text{memdim}(S) \geq \text{diam}(G)$

Routing in a graph as path

- Lemma 4:

If G is a path, then there exists an S s.t. (G, S) is shattered and internally connected with memdim(S) = diam(G)



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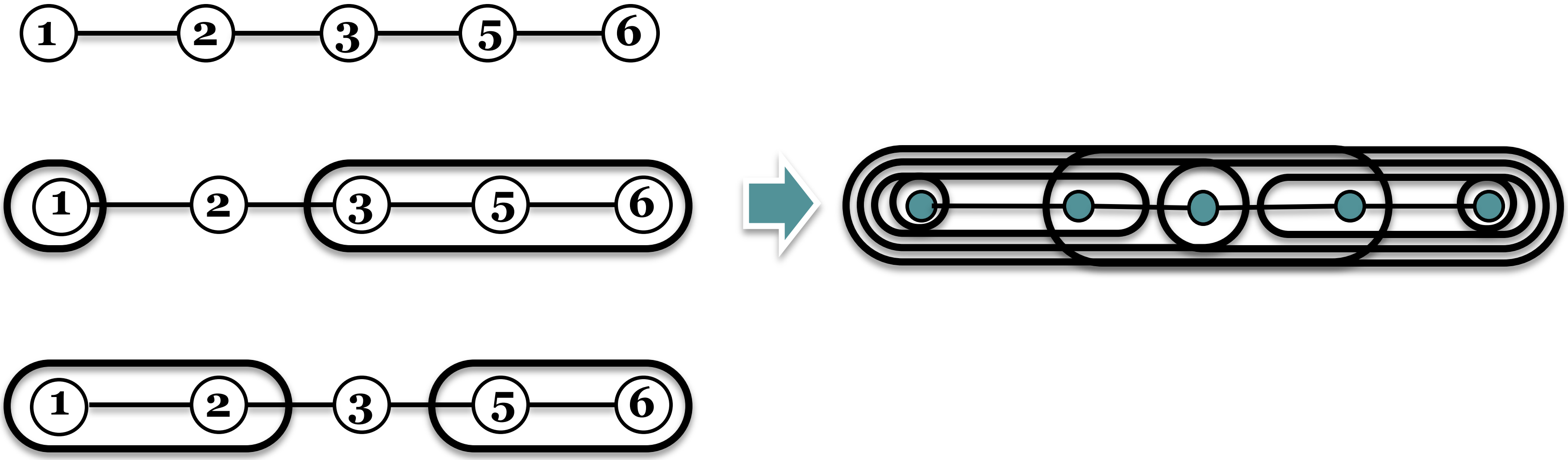

Routing in a binary tree

- Lemma 5:

If G is a binary tree, then there exists an S s.t. (G,S) is shattered and internally connected with: $\text{memdim}(S) = O(\text{diam}^2(G))$

Why $\text{memdim}(S) = O(\text{diam}^2(G))$?

For $v \in U$: $u \in \text{ancestors}(v)$

$\Rightarrow v \in S_u$ and v belongs to O(height(u)) sets of L_u and R_u

$\Rightarrow$ v belongs to $O(\sum_{u \in \text{ancestor}(v)} \text{height}(u))$ sets

$\Rightarrow O(\text{diam}^2(G))$

Converting a n-node rooted tree

- Lemma 6:

Let T be an n-node rooted tree with height h. We can embed T into a binary tree such that the ancestor-descendant relationship is preserved, and the resulting tree has a height $O(h + \log n)$

Upper bound of the cognitive load

Theorem:

If G is connected, there exists S s.t. Routing works and memdim(S) $= O((\text{diam}(G) + \log n)^2)$

- Compute a low-diameter spanning tree T of G $\xrightarrow{\text{BFS}}$ $\text{diam}(T) \leq 2\text{diam}(G)$
- Arbitrary root T and embed T into a binary B

  with height $O(\text{diam}(T) + \log n)$, by Lemma 6
- $\text{diam}(B) = O(\text{diam}(T) + \log n)$
- By Lemma 5 $\Rightarrow$ $\text{memdim}(S_B) = O((\text{diam}(T) + \log n)^2)$
- From S_B to S_T and $\text{memdim}(S_T) \leq \text{memdim}(S_B) = O((\text{diam}(T) + \log n)^2)$

$$\Rightarrow \text{memdim}(S\ ) = O((\text{diam}(G) + \log n)^2)$$

Summary

- Arbitrary graph: memdim(S) $\geq$ diam(G)
- Path: memdim(S) = diam(G)
- Binary tree: $\text{memdim}(S) = O(\text{diam}^2(G))$
- From arbitrary to binary tree: height of $O(h + \log n)$
- Arbitrary graph: $\text{memdim(S)} = O((\text{diam}(G) + \log n)^2)$

Thank you!