How Optimal are Wireless Scheduling Protocols?

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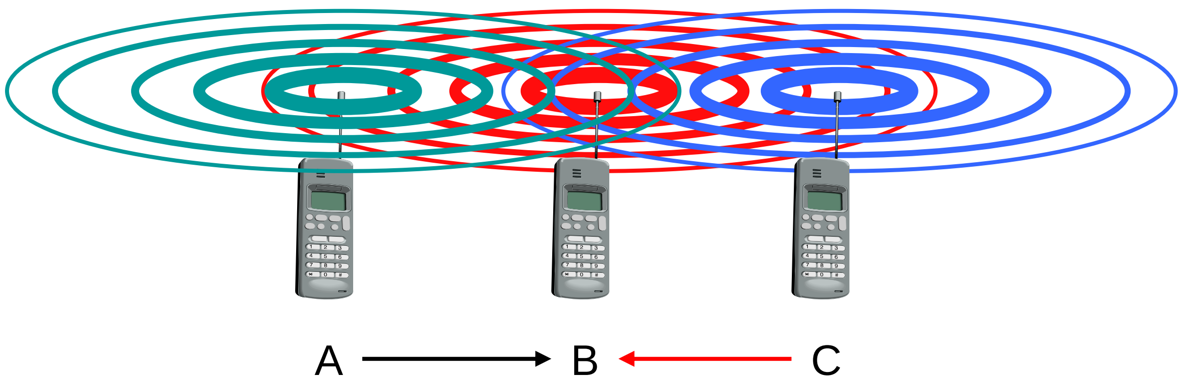
Scheduling in Wireless Networks

Scheduling is crucial in wireless networks.

- Modeling interference in a typical textbook or algorithms paper:



- A lot of existing work on the impact of interference
- But, are the foundations (possibilities, limitations) of MAC layers really understood…?
- Do we have competitive scheduling (MAC layer) protocols?

A Simple Scheduling Problem

Consider the following scheduling task Λ

Λ: set of communication requests $\lambda_i=(s_i, r_i)$

sender s_i, receiver r_i coordinates in Euclidean plane

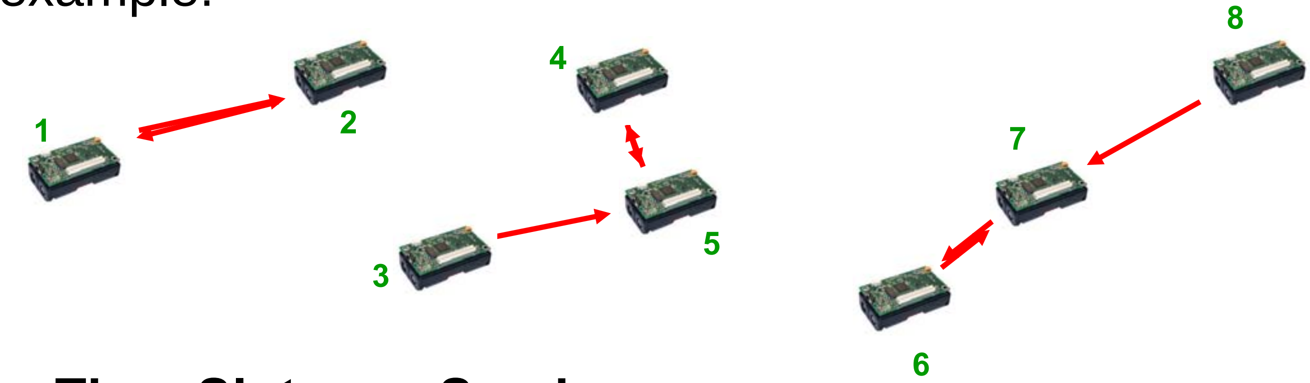
How many time-slots are required so every request is scheduled?

„The Scheduling Complexity in Wireless Networks"

Compare to *capacity of wireless networks*…

A Simple Scheduling Problem - Example

An example:



Time-Slot	**Senders:**
t_1:	v_1, v_4, v_7
t_2:	v_2, v_3, v_6
t_3:	v_5, v_8

This scheme uses 3 time slots!

→ **Scheduling complexity of Λ is 3 in this example.**

What is the scheduling complexity of wireless networks as a function of n? (scaling laws)

Physical Model

- Let us look at the signal-to-noise-plus-interference (SINR) ratio!
- Message arrives if SINR is larger than β at receiver

Received signal power from sender

Power level of sender u

Path-loss exponent

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N + \sum_{w \in V \setminus \{u\}} \frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

Minimum signal-to-interference ratio

Noise

Distance between two nodes

Received signal power from all other nodes (=interference)

Related Work

- There is a lot of related work on scheduling
  → numerous practical scheduling protocols
  → wireless MAC layer protocols

- Capacity of wireless networks [Gupta, Kumar, Trans.Inf.Theory'00]

- Combined power assignment and scheduling problems
  [Behzad, Rubin, Infocom'05], [Jain, Padhye, Padmanabhan, Qiu, Mobicom'03], [Bjorklund, Varbrand, Yuan, Infocom'03], etc…

- Specifically SINR based scheduling protocols
  [Ephremides,Truong,Trans.Comm'90], [ElBatt, Ephremides, Infocom'02], [Cruz, Santhanam, Infocom'03], etc…

- Comparison between graph-based and SINR-based scheduling
  [Gronkvist, Hansson,Mobihoc'01], etc…

Competitive scheduling protocol...?

Scheduling complexity in wireless networks...?

The Scheduling Complexity of Wireless Networks

- n nodes in 2D Euclidean plane
  - Nodes can choose power levels
  - Message successfully received if SINR at receiver sufficient

Scheduling Complexity $S(\Lambda)$

The minimal number of time-slots required until every request is successfully transmitted (in every network)!

Clearly, $S(\Lambda) \leq n$

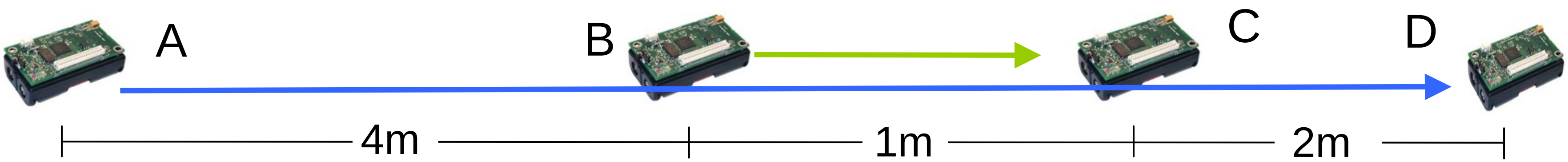
Graph-based models...? → Coloring, independent sets...

SINR: Fixed power assignment...? → Mathematical optimization

SINR: Free power assignment...?

Example: Graph-based vs. Physical Model

A sends to D, B sends to C



Assume a single frequency (and no fancy decoding techniques!)

Is spatial reuse possible?

- NO — Graph-based Models
- YES — Physical Model
  - In Reality!

Let α=3, β=3, and N=10nW

Transmission powers: P_B= -15 dBm and P_A= 1 dBm

SINR of A at D: $$\frac{1.26mW/(7m)^3}{0.01\mu W+31.6\mu W/(3m)^3} \approx 3.11 \geq \beta$$

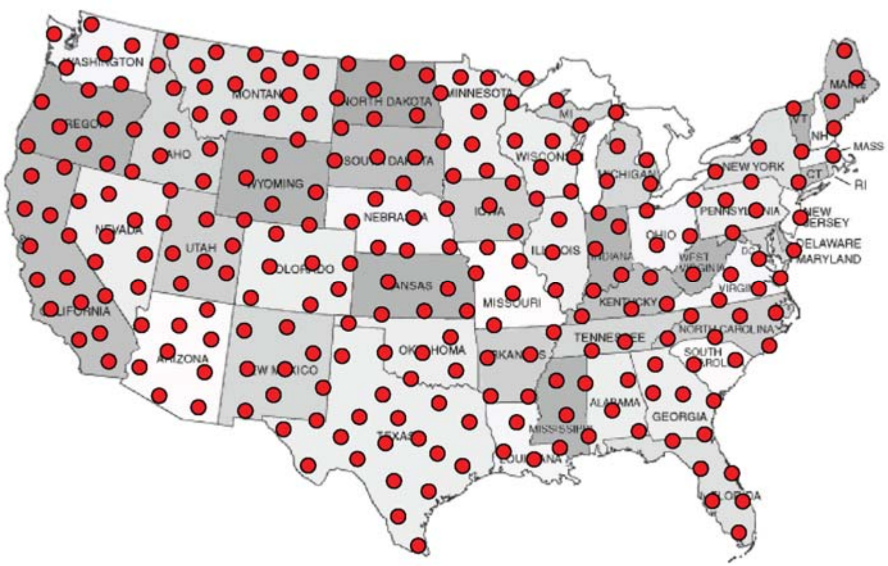
SINR of B at C: $$\frac{31.6\mu W/(1m)^3}{0.01\mu W+1.26mW/(5m)^3} \approx 3.13 \geq \beta$$

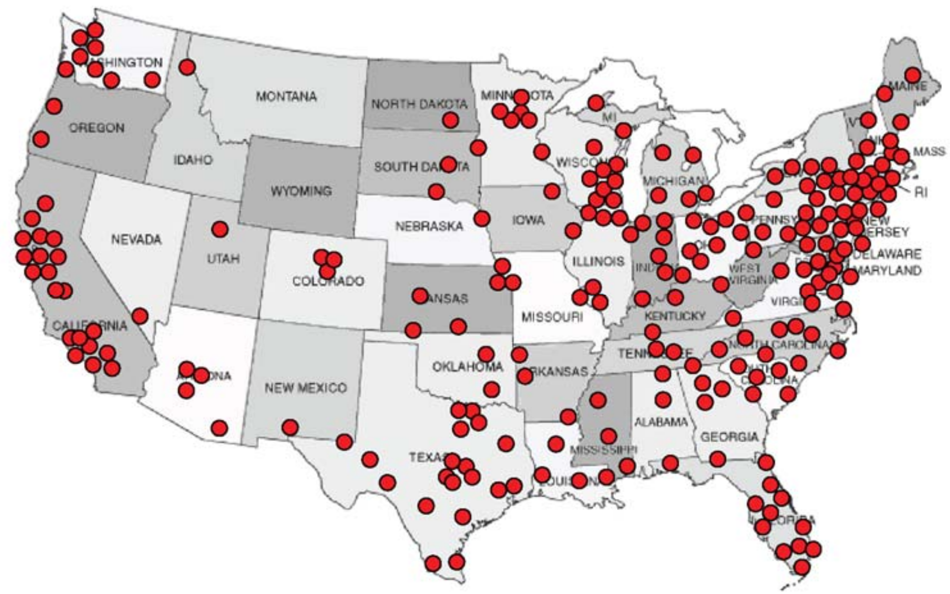
[Moscibroda, Wattenhofer, Weber, Hotnets'06]

Worst-Case Complexity

- Previous work: heuristics or very strong assumptions on node deployment, topologies
  - randomly, uniformly distributed nodes
  - nodes placed on a grid
  - etc...

What do real networks look like?





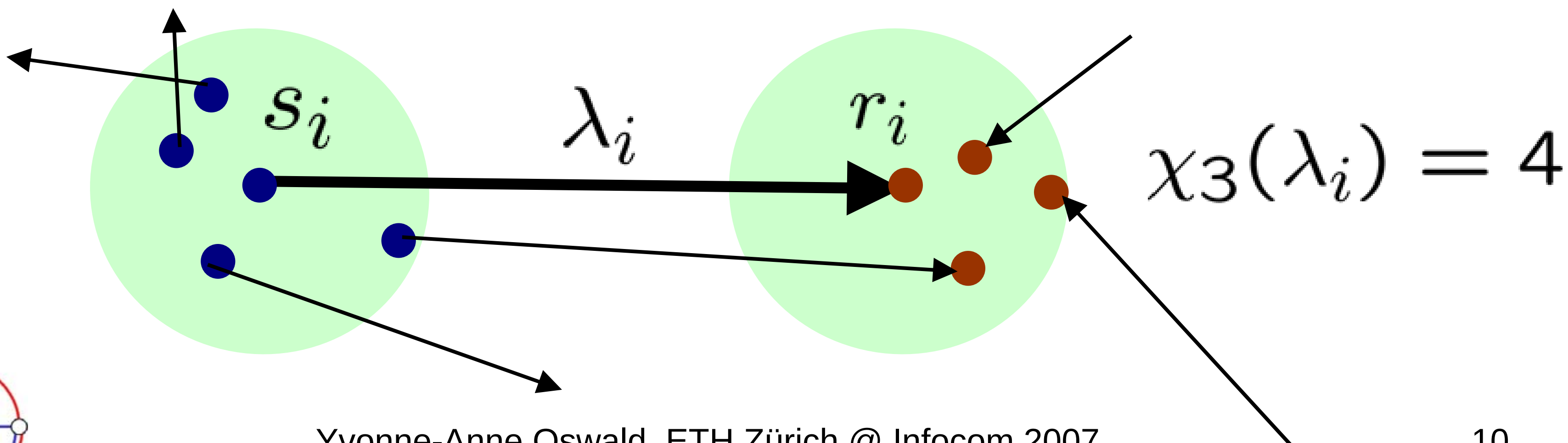
ρ - Disturbance

- Measure for intrinsic difficulty of scheduling task

ρ-disturbance for a given set Λ

$$\chi_\rho := \max_{\lambda_i \in \Lambda} \chi_\rho(\lambda_i),$$

where $\chi_\rho(\lambda_i) = \max(|\{r_j \mid d(r_j, r_i) \leq d_i/\rho\}|, |\{s_j \mid d(s_j, s_i) \leq d_i/\rho\}|)$.



Power Assignment Policies

Commonly employed/assumed power assignment policies

- All nodes have uniform power
  - Implicit assumption in unit disk graph
- Powers are according to
  - This linear power assignment often (implicitly) assumed $P \sim d^{\alpha}$ (e.g., energy metric, topology control, etc...)
- Any kind of fixed power scheme
  - power as a function of link length
  - A discrete set of power levels

Schedule of length $\Omega(n)$ even in scenarios with low ρ-disturbance **[Moscibroda et al., Infocom 06]**

Link Removal Algorithms

postpone links according to some condition
until SINR>β for remaining links

- SRA (Stepwise Removal Algorithm), [Zander, 92] : remove link with largest row or column sum of inverted gain matrix

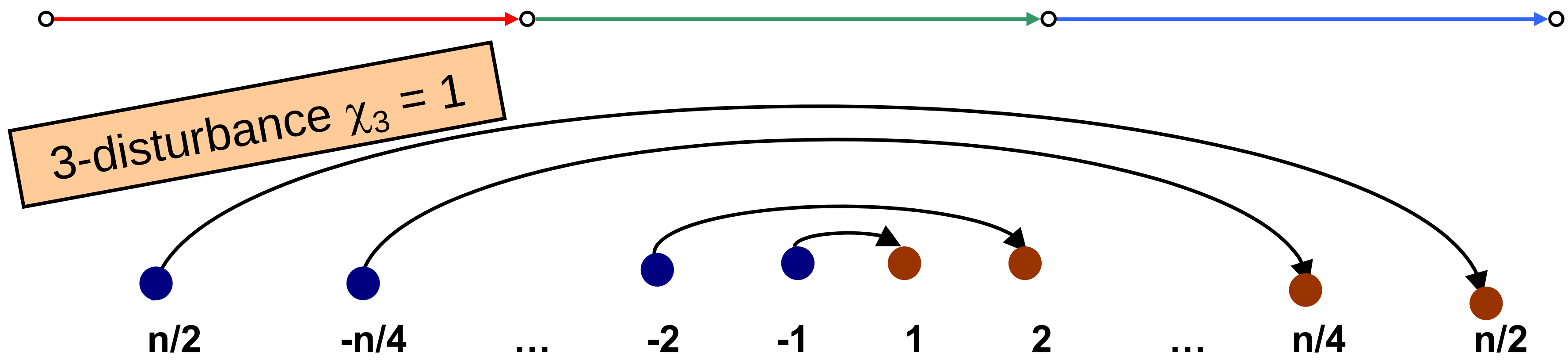
- SMIRA (Stepwise Maximum Interference Removal Algorithm), [Lee et al., 95], exclude links causing or receiving most interference under optimal power assignment

- WCRP [Wang et al., 05](distributed) remove links with MIMSR>ζ

$$MIMSR(i) = \max_{j \neq i}\{\frac{\beta G(i,j)}{G(i,i)}\}$$

- LISRA (Limited Information Stepwise Removal Algorithm),[Zander,92], postpone link with the lowest *SINR* if equal sending power

Worst Case Complexity



- Schedule length in Ω(n) for SRA, SMIRA, WCRP, LISRA

- Schedule sets $\{\lambda_t, \lambda_{t+\log n}, \lambda_{t+2\log n}, \ldots\}$, log *n* slots necessary
  For link $\lambda_i = (-2^i, 2^i)$, the $\tau(i)^{th}$ longest link: $P(s_i) = (2n)^{\tau(i)}\, 2^{\alpha(i+1)}$

$\Rightarrow$ Short links send with more power than necessary to reach receiver

LDS- Protocol

- Novel power assignment
- New spatial reuse criteria

3 parts:

a) Pre-processing phase

b) Main scheduling loop

c) Subroutine allowed

Pre-processing phase:

$\tau\mathrm{cur} := 1$; $\gamma\mathrm{cur} := 1$; $\mathrm{last} := d_1$;
Consider all requests $\lambda_i \in \Lambda$ in decreasing order of d_i:
for each $\lambda_i \in \Lambda$ **do**
  if $\mathrm{last}/d_i \geq 2$ **then**
    if $\gamma\mathrm{cur} < \lceil \log(3n\beta) + \rho \log \alpha \rceil$ **then**
      $\gamma\mathrm{cur} := \gamma\mathrm{cur} + 1$;
    else
      $\gamma\mathrm{cur} := 1$; $\tau\mathrm{cur} := \tau\mathrm{cur} + 1$;
    end
    $\mathrm{last} := d_i$;
  end
  $\gamma(i) := \gamma\mathrm{cur}$; $\tau(i) := \tau\mathrm{cur}$;
end

Main scheduling-loop:

Define constant ν such that $\nu := 4N$;
$t := 1$;
for $k = 1$ **to** $\lceil \log(3n\beta) + \rho \log \alpha \rceil$ **do**
  Let $\mathcal{F}_k$ be the set of all requests λ_i with $\gamma(i) = k$.
  while not all requests in $\mathcal{F}_k$ have been scheduled **do**
    $L_t := \emptyset$;
    Consider all $\lambda_i \in \mathcal{F}_k$ in decreasing order of d_i:
      if $allowed(\lambda_i, L_t)$ **then**
        $L_t := L_t \cup \{\lambda_i\}$; $\mathcal{F}_k := \mathcal{F}_k \setminus \{\lambda_i\}$
      end if
    Schedule all $\lambda_i \in E_t$ in time slot t, assigning s_i a transmission power of $P_i = \nu \cdot d_i^{\alpha} \cdot (3n\beta\rho^{\alpha})^{\tau(i)}$;
    $t := t + 1$;
  end while
end for

$\mathrm{allowed}(\lambda_i, L_t)$

Define constant μ such that $\mu := 4\sqrt[\alpha]{\frac{120\beta(\alpha-1)}{\alpha-2}}$;
for each $\lambda_j \in L_t$ **do**
  $\delta_{ij} := \tau(i) - \tau(j)$;
  if $\tau(i) = \tau(j)$ and $\mu \cdot d_i > d(s_i, s_j)$
  or $\tau(i) > \tau(j)$ and $d_i \cdot (3n\beta\rho^{\alpha})^{\frac{\delta_{ij}+1}{\alpha}} > d(s_i, r_j)$
  or $\tau(i) > \tau(j)$ and $d_j/\rho > d(s_j, r_i)$
  then return false
end for
return true

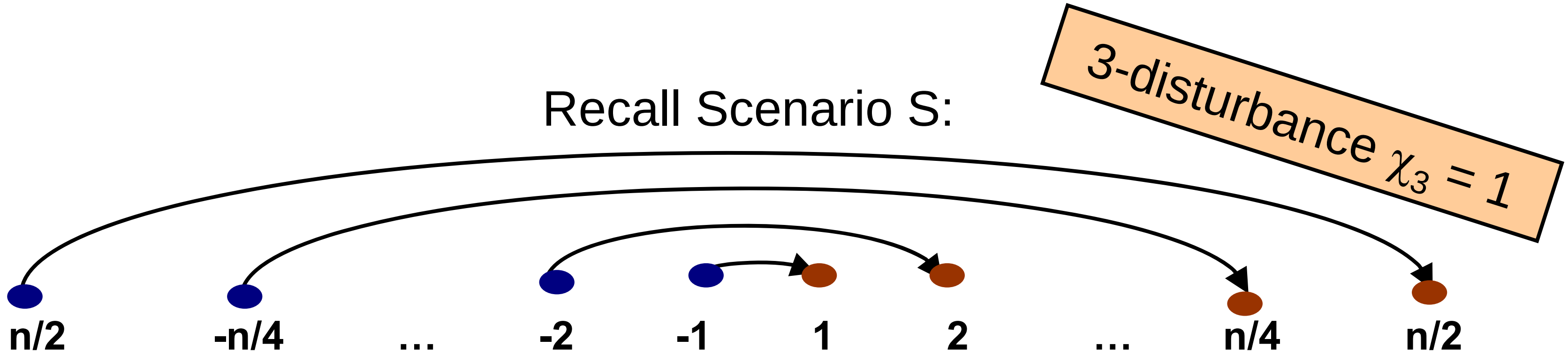
Performance of LDS

- Results in correct schedule: all communication requests can be transmitted successfully

(due to χ_ρ the maximal interference can be bounded)

- Length of schedule: $O(\chi_\rho\ \rho^2 \log n\ (\log n + \rho))$

Recall Scenario S:



•Schedule length in $\Omega(n)$ for SRA, SMIRA, WCRP, LISRA

•LDS O(log2 n)

Conclusions

- ρ - disturbance measures intrinsic difficulty of scheduling complexity
- Existing protocols heuristics or only efficient if nodes randomly distributed, no worst case guarantees
- LDS polylogarithmic scheduling complexity for low ρ-disturbance

- Better understanding of scheduling in wireless networks, improvements for MAC layer?

- Open question: better algorithms possible? For any ρ? Distributed?

That's it…

THANK YOU!